ANALYSIS OF REALIZED VOLATILITY ESTIMATORS OF EUROPEAN STOCK INDICES

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DOKTORSKA DISERTACIJA

ANALIZA PROCJENITELJA REALIZIRANE VOLATILNOSTI EUROPSKIH BURZOVNIH INDEKSA

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UNIVERSITY OF SPLIT
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DOCTORAL THESIS

ANALYSIS OF REALIZED VOLATILITY ESTIMATORS OF EUROPEAN STOCK INDICES

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Predgovor, posveta i sažetak na hrvatskom jeziku

Predgovor i posveta

Ova doktorska disertacija je za mene mnogo više od znanstvenog istraživanja. Ovo je bilo jedno putovanje, vođeno znanošću i radoznalošću, koje je započelo mnogo godina prije nego što sam bio primljen na doktorski studij na Ekonomskom sveučilištu u Splitu. Vrativši se u 1996. godinu, upravo sam tijekom studija ekonomije u Dubrovniku upoznao umjetnost ekonometrije. U to vrijeme sam neprestano tražio konkretne odgovore na mnoga pitanja na području ekonomije nesluteći da se u meni jednim dijelom skriva „kvantitativac". Vođen znatiželjom, podnio sam prijavu na sveučilištu Erasmus u Rotterdamu za diplomski studij ekonometrije i menadžmenta. Nekoliko mjeseci kasnije predao sam diplomski rad u Dubrovniku i ubrzo diplomirao na Ekonomiji. Po primitku diplome sveučilišta u Rotterdamu, započeo sam raditi u financijskoj industriji gdje sam kao mladi kvantitativac mogao primjeniti stečena znanja. Uz rad sam nastavio magistarski studij kvantitativnih financija na istom sveučilištu. Nakon magistarskog studija ekonometrije i menadžmenta bio sam u potpunosti opremljen najnovijim znanstvenim spoznajama iz područja kvantitativnih financija. Stečeno radno iskustvo u međunarodnim financijskim tvrtkama u Nizozemskoj odlučio sam primjeniti u Hrvatskoj gdje je moj akademski put i započeo. Profesionalnu karijeru nastavio sam u bankarskom sektoru kao menadžer korporativnih kreditnih rizika. Iako sam već imao nekoliko fakultetskih diploma, moja se žeđ za znanstvenim istraživanjima nije ugasila i tražio sam nove izazove. Ekonomski fakultet Sveučilišta u Splitu ponudio mi je mogućnost doktorskog studija na području kvantitativne ekonomije.

Tijekom prve tri godine istraživačkog programa Ekonomskog fakulteta obogatio sam i proširio svoje znanje vrlo intenzivnim kolegijima iz područja ekonomije. Doktorski studij zahtijevao je visok stupanj predanosti, osobnu inicijativu i samostalno učenje, koje sam nastavio tijekom cijelog studija. Važno je spomenuti i neočekivani životni put koji me je već za vrijeme studija selio u sedam različitih gradova i dvije različite zemlje Europe. S obzirom da spomenute promjene mjesta studija, rada i samog boravka oduzimaju vrijeme, planirani put doktorskog

studija se odužio i dodao je novi i neočekivani izazov. Ipak sam uvijek sa sobom nosio svoje istraživanje i nastavljao gdje sam stao.

Veoma sam zahvalan svom mentoru dr. sc. Josipu Arneriću na smjernicama i pruženoj mogućnosti provedbe doktorskog istraživanja o modelima volatilnosti finacijskog tržišta na temelju raspona i njihovoj primjeni u europskim burzama. Upravo zahvaljujući trudu dr. sc. Josipa Arnerića dobili smo pristup Reutersovoj intradnevnoj bazi podataka koja je postala osnova empirijskog istraživanja ovo rada.

Posebno zahvaljujem svojoj supruzi koja me podržavala tijekom cijelog studija. Znam da ti nije uvijek bilo lako prihvatiti da slobodno vrijeme, uključujući i one dragocjene vikende, provodim radeći na svojoj disertaciji. Mnogo puta smo žrtvovali lijepe, ali i rijetke trenutke toplog i sunčanog vremena u Nizozemskoj da bih se posvetio svome radu. Svjestan sam da je u jednom trenutku i tebi postao izazov slušati priče o procjeni volatilnosti pomoću intradnevnih podataka. Na kraju, tvoja potpora i motivacija su one koje su me gurale naprijed i pomogle da svoj rad privedem kraju. Podržavala si me u najtežim trenucima mog života i nanovo me motivirala bezbroj puta, bez tvoje pomoći ovo ne bih ostvario.

Ovaj rad posvećujem svom sinu, Eliu, koji mi omogućuje da nanovo vidim kreativnosti svijeta i svojoj supruzi, Ireni, koja je moja prekrasna partnerica u pustolovini života.

SAŽETAK DOKTORSKE DISERTACIJE

Ova disertacija istražuje opažanja intradnevnih cijena šest izranjajućih europskih dioničkih tržišta te empirijskom analizom istražuje učinkovitost rasponskih procjenitelja volatilnosti. Teorija realizirane volatilnosti koristi intradnevne podatke za procjenu integrirane volatilnosti. Međutim, realizirana volatilnost je često pristrana zbog raznih mikrostrukturnih efekata. Glavni procjenitelj u ovom radu je nepristrani procjenitelj integrirane volatilnosti koji procjenjuje volatilnost u dvije „faze“, te se razlikuje od realizirane volatilnosti. U radu se koristi skraćenica engleskog naziva „Two Times Scale Estimator“, odnosno TTSE, kako bi se označio nepristrani procjenitelj integrirane volatilnosti. Za svaki od šest burzovnih indeksa određen je odgovarajuć i asimptotski nepristran procjenitelj integrirane volatilnosti koristeći TTSE procjenitelj na bazi tržišnih cijena promatranih visokom intradnevnom frekvencijom. Bitno je naglasiti da sama opažanja intradnevnih cijena pružaju dodatan i veoma vrijedan uvid u cjenovne promjene tijekom sati trgovanja na burzi. Međutim, postoje određena ograničenja kod široke primjene u cijeloj financijskoj industriji budući da intradnevni podaci nisu uvijek dostupni za svako tržište odnosno za svaki financijski instrument. Razlozi potonjeg variraju od ograničenih podataka do nelikvidnih tržišta. Kao razumnu alternativu za razne primjene u području financija, ova disertacija sugerira korištenje rasponske procjenitelje volatilnosti koji koriste samo ograničen broj opažanja intradnevnih cijena, tj. cijena otvaranja, najviša cijena, najniža cijena i cijena zatvaranja tijekom dana trgovanja. U radu se koristi skraćenica engleskog naziva „open, high, low, close” (OHLC).

Standardni rasponski procjenitelji volatilnosti ne uzimaju u obzir informaciju koja se promatra van standardnih sati trgovanja. Promjene u cijenama koje se realiziraju van standardnih sati trovanja nazivaju se „prekonoćnim skokovima“. Prekonoćni skokovi u cijenama promatraju se kao razlika između početne cijena trenutnog dana i završne cijene prethodnog dana. U radu su standardni rasponski procjenitelji obogaćeni prekonoćnim skokovima te s ovim rad doprinosi već bogatoj literaturi o procjeniteljima financijske volatilnosti. Drugi dio ovog rada usredotočen je na izazov rangiranja rezultata procjenitelja volatilnosti. Koliko je autoru ovog rada poznato, literatura nije bila jednoglasna u metodologiji rangiranja ovih procjenitelja, te smatra da metoda rangiranja uvelike utječe na konačan optimalan izbor procjenitelja. Ovo istraživanje uspoređuje nekoliko metodologija rangiranja te nastoji ponuditi smjernice u izboru metodologije rangiranja s obzirom na svoju svrhu. Postojeće metodologije rangiranja (kao što su funkcija gubitka, koeficijent učinkovitosti i Mincer Zarnowitzeva regresija) usredotočene su na cjelokupnu statističku raspodjelu, dok su ekstremne promjene u volatilnosti cijena često nedovoljno pokrivene. Ovo istraživanje koristi koeficijent gornje ovisnosti repa (eng. „upper tail-dependence“), koji je rezultat Gumbelove Copula funkcije, za svrhe usporedbe kada su fokus interesa upravo sami repovi raspodjele. Zavisnost u gornjem dijelu repa raspodjele smatra se komplementarnom metodologijom rangiranja koja uz standardnu funkciju gubitka ili uz pristup koeficijenta učinkovitosti, upotpunjava analizu rangiranja procjenitelja volatilnosti. Rezultati pokazuju da su modeli volatilnosti na bazi raspona prikladne alternative procjenitelju TTSE i da ni standardna devijacija ni dnevni kvadratni prinos, koji se među standardnim rasponskim procjeniteljima volatilnost smatraju najpopularnijim, nisu odabrani u bilo kojoj metodologiji rangiranja. Među izabranim rasponskim procjeniteljima volatilnosti su Parkinson, Garman-Klass, High-Low, Roger-Satchell te Yang-Zhang. Optimalan model rasponske volatilnosti ovisi o primjenjenoj metodi rangiranja, odnosno o samoj primjeni u praksi.

Ključne riječi: *Integrirana volatilnost, realizirana varijanca, rasponski procjenitelji volatilnosti, OHLC, funkcija gubitaka, koeficijent efikasnosti, izranjajuće dioničko tržište.*

Foreword, dedication and Summary in English

Foreword and dedication

This PhD study has been much more than a scientific research on its own. It has been a journey driven by science and curiosity that has started many years before I was accepted as a PhD candidate at the Economic University of Split. Going back to 1996, it was during the study of Economics in Dubrovnik that I became acquainted with the art of Econometrics. Back then I didn't know yet that there was a hidden "math personality" in me as I was continuously searching for concrete answers to the many questions that were asked in the field of Economics. Beaten by my curiosity I submitted an application at the Erasmus University in Rotterdam for a Bachelors in Econometrics and Management Science. This happened even before graduating at the Economic Faculty in Dubrovnik. A few months later I submitted my Thesis in Dubrovnik and graduated soon in Economics, but was even more fascinated with the exact science that I was given the opportunity to master. After receiving my Bachelor I started working in the financial industry where I, as a young Quant, applied the learned knowledge directly in the field. Alongside I continued with a subsequent Master's study in Quantitative Finance at the same University. After receiving my Master's degree in Econometrics and Management Science I was fully equipped with the most recent scientific knowledge in the field of quantitative finance. Together with the gained working experience in international financial companies in the Netherlands I decided to test my knowledge and experience back in Croatia where it all started. I continued my professional career in a retail bank as a Risk Manager. Although I already had several University degrees, my thirst for scientific studies didn't quench. I became even more curious and was seeking for a new challenge in life. The Faculty of Economics at the University of Split offered me the opportunity for a PhD study in Economics. This life time opportunity at this University was a challenge I gladly accepted.

During the first three years of the research-intensive program at the Faculty of Economics I have enriched and broadened my knowledge with various highly intensive courses in the field of Economics. The PhD program required a high degree of commitment, personal initiative and self-directed learning, which I have continued during the entire study. Important to mention was an unexpected life changing development, which has forced me to move in seven different cities and two different countries across Europe during my study. This time consuming process has delayed the planned progress sincerely and has added a new and unexpected challenge in the study. Nevertheless I have always carried my research with me and continued from where I stopped.

I am heavily indebted to my promoter and supervisor, Josip Arnerić, for his guidance and for giving me the opportunity to conduct the PhD research on range-based financial market volatility models and their application in European stock markets. It was thanks to his effort that we gained access to the Reuters intraday database, which has become the basis for the empirical research provided in this work.

I would like to specially thank my wife who supported me throughout the study. I know it wasn't always easy for you knowing that I would spend our free time, including those precious weekends, working on my Thesis. Many times we have traded in those beautiful, yet rare moments of warm and sunny weather in the Netherlands for this Thesis. I also know that at some point it became a challenge for you as well to listen to the stories about estimating volatility using intraday data. In the end, it was your support and motivation that kept me going and finalizing this work. You have supported me during the toughest moments in my life and motivated me for countless times. Without your help this wouldn't have succeeded.

I dedicate this work to my son, Elio, who enables me to see the creativities of the world anew and to my wife, Irena, who is my beautiful partner in the adventure of life.

SUMMARY OF THE THESIS

This Thesis investigates intraday price observations for six European emerging stock markets and explores the effectiveness of range-based volatility estimators with an empirical analysis. The theory of Realized Volatility utilizes intraday data to estimate the integrated volatility. However, Realized Volatility is often biased due to microstructure noise. The Two Times Scale Estimator is an unbiased estimator of the integrated volatility.

For each of the indices a consistent and asymptotically unbiased estimator of the integrated volatility is determined using intraday price observations and the Two Times Scale Estimator. As intraday price observations provide additional insight in the price changes during trading hours, it also has limitations in industry wide applicability as intraday data is not always available. The reasons can vary, from restricted data to illiquid markets. As a reasonable alternative for many applications in finance this Thesis suggest the use of range-based volatility estimators that utilize only a limited number of intraday price observations, i.e. the Open, High, Low and Close price observations. The standard range-based volatility estimators are extended with the information captured in overnight jumps and contribute to the already rich literature on financial volatility.

The second part of this Thesis focuses on the challenge of ranking the results. As far as we are aware the literature had not been unanimous on the ranking methodology. This research compares several ranking methodologies and attempts to provide more guidelines in the choice of the ranking methodology given its purpose. The existing ranking methodologies (loss functions, coefficient of efficiency and the Mincer Zarnowitz regression) focus on an overall fit, while the extreme movements are often insufficiently covered. This research employs the upper tail dependence coefficient, a result of the Gumbel copula function, for comparison purposes when the focus of interest are the tails of the distribution. The upper tail dependence is a complementary ranking methodology to the standard loss functions or coefficient of efficiency approach.

The results show that range-based volatility models are appropriate alternatives to the Two Times Scale Estimator and that the neither the standard deviation nor the daily squared return have been selected in any of the ranking methodologies.

Keywords: Integrated volatility, Realized variance, range-based volatility estimator, OHLC, Loss function, Coefficient of Efficiency, Upper tail dependence, emerging market.

"*Financial volatility is unobservable. Only the realized changes in asset returns convey some information about what volatility actually is*" (Knight, J. and Satchell, S., 2007).

MOTIVATION

Volatility of financial markets is a key element in financial modelling and forecasting. It is commonly referred to as a measure of risk or a measure that represents risk. The risk usually increases with the volatility, i.e. the higher the volatility, the higher is the perception of risk in the model. Volatility itself is not a variable that van be observed and therefore requires a methodology to quantify it. The information that is captured in the realized price changes in asset returns convey valuable information about the volatility. Many models in the literature utilize this price information to determine the volatility. There are many models to choose from, yet the differences in performance between these models can be significant. The consequence of a wrong choice could adversely affect the financial model or the forecast and, hence, the decisions based upon these results. Therefore the choice of the volatility model is of key importance for financial modelling and has been a key topic in scientific research in the past decades.

The existing financial literature has developed a vast amount of models for quantifying volatility, which range from realized volatility models to GARCH-like models. However, for as far we are aware of, the literature has not been successful in being unanimous on the optimal volatility model or the methodology of ranking these models. The consequence is that there are no guidelines for practitioners when choosing the most appropriate model from a large set of available models. Recent market developments that have marked the availability of high frequency intraday data, for instance when price changes are observed with a frequency of a second, have also contributed to an increase in new volatility models in the literature. The availability of high frequency intraday data made it possible to answer questions on intraday price movements for a vast number of securities across different markets and asset classes. This development has given a boost to the theory of realized volatility, which states that an important result of properly using the available intraday data can result in an unbiased estimator of the true volatility.

An important downside of working with high frequency data are the numerous challenges with high frequency data, which in some cases may limit the usability of the theory of realized volatility. For instance, high frequency data is only available for markets and asset classes that have intraday trading activities. Hence, it is not possible to calculate the realized volatility for illiquid asset classes. The applicability of intraday price information for modelling purposes depends not only on the existence of intraday trading activities, but also on their volume and liquidity. In this context many Emerging markets have been lacking behind the facts compared to their Western counterparts. The volume of intraday trades can be simply insufficient for model usage. As a result, volatility models that utilize high frequency price information of a particular market or industry are limited by the availability and quality, measured by its volume and liquidity, of the required intraday data. Another limitation is that high frequency data also includes working with extensive datasets that may require state of the art data crunching challenges. Although this is not strictly impossible, having the recent technological developments in scope, it remains reasonable to take the cost-effectiveness of this additional, time-consuming effort into account.

For the reasons outlined we find that volatility models that utilize the information extracted from low frequency datasets, for instance using a limited number of intraday price observations, offer certain advantages compared to those models that utilize information extracted from high frequency datasets. For instance, when analysing the price history of financial markets or asset classes with a poor intraday trading volume on the high frequency levels. In this case the theory of realized volatility would not be suitable to estimate the volatility as there is insufficient intraday price observation at the high frequency level. This is a common problem in many emerging markets and in less liquid asset classes. Also in many other cases one might consider using a more simplistic but efficient volatility model compared to the complex volatility models that utilize a vast amount of high frequency trades and require additional investigation time and a more complex model.

A reasonable alternative to using complex volatility models based on high frequency intraday data would be to use volatility estimates that are based on a limited number of intraday price observations that are at the same time widely available for a large set of asset classes and markets. Many of this type of volatility models have been developed in the mid-80s, but have been left in the shadow of the complex volatility models based on GARCH and the high frequency intraday price observations. This is somewhat justified as the application of these low frequency models has remained limited in the literature. It has also been noticed that the rich literature of GARCH models has left the simplistic volatility models in the shadow. As far as we are aware one of the main disadvantages of the literature on low frequency volatility models is that the efficiency comparison was theoretical as there was no proper benchmark to compare the results with. The availability of high frequency data made it possible to compare the performances of alternative volatility models with the unbiased volatility model in a back-test situation.

The results of the volatility models are compared against the benchmark, i.e. the unbiased intraday volatility estimator. The literature has mentioned several methods for ranking the volatility models, but has not been unanimous on the ranking methodology itself. We are also not aware of any guidelines for users on which ranking methodology to use and in which situation. For instance, one might be interested in a simple volatility model that has an overall good performance across the entire distribution or one might be interested in a simple volatility model that efficiently measures the volatility in the tails of the distribution. In most cases the ranking methodology doesn’t cover both questions and, hence, a combination of different ranking methodologies is required.

The motivation of this Thesis is threefold. First this research focuses on estimating the unbiased volatility by utilizing high frequency intraday price observations and on finding the alternative volatility estimates that avoid the limitations and challenges of high frequency price observations when necessary. This research advocates models that utilize only a limited subset of the intraday interval without significant loss of efficiency in estimating the stock market volatility. The open, high, low and closing (OHLC) prices are available for a wide range of securities in multiple markets and asset classes, which is not always the case for high frequency data. This research paper proposes to use intraday prices from the OHLC prices dataset for volatility estimation. Secondly, this research uses several ranking methodologies that have been previously used in the literature to rank the volatility models. This Thesis also suggests a new ranking methodology that is based on the tail correlation as a function from a Copula function to rank the volatility estimates based on their performance in the tail of the distribution. Thirdly, this research provides various financial applications where volatility models based on low frequency price observations are used to estimate the volatility.

CONTENTS

LIST OF TABLES

List of Figures

LIST OF APPENDICES

List of Abbreviations and Acronyms

BV	Bipower Variation
CC	Close to close volatility estimator
CC*	Conditional Coverage test
CO	Close to open volatility estimator
COC	Close to open to close volatility estimator
EF	Efficient Frontier
EWMA	Equally Weighted Moving Average
Ext	Extended model
GARCH	Generalized Auto-Regression Conditional Heteroscedasticity
GBM	Geometric Brownian Motion
GK	Garman Klass volatility estimator
HL	High Low volatility estimator
HL Ext	High Low Extended volatility estimator
HF	High Frequency
IND	Independence test
IV	Integrated Volatility
LPM	Lower Partial Moment
MAE	Mean Absolute Error
MPT	Modern Portfolio Theory
MSE	Mean Square Error
OHLC	Open High Low Close
OC	Open to close volatility estimator
Park	Parkinson volatility estimator
Park Ext	Extended Parkinson volatility estimator
QLIKE	Quasi Likelihood
RMSE	Root Mean Square Error
RS	Roger Satchell volatility estimator
RS Ext	Roger Satchell Extended volatility estimator
RV	Realized Volatility
TTSE	Two Time Scale Estimator
UC	Unconditional Coverage test
VaR	Value at Risk

1 INTRODUCTION

1.1 Subject of the research

This research analysis the performance of various range-based volatility estimators and compares the results against an unbiased benchmark estimator. The analysis consists of an empirical research, which is performed on a number of East European stock markets. The East European stock markets are represented by the main stock market indices of Bulgaria, Czech Republic, Croatia, Hungary, Poland and Romania. The historical data includes intraday price observations recorded at a 1-minute interval, which can be considered an extremely high frequency for the markets under consideration. The number of (intra-)daily trades in East European stock markets is, generally speaking, much less than those in the highly developed markets, where stock indices can often be observed on a tick-by-tick basis. For example, S&P 500, Dow 30, NASDAQ, etc. The analysis is performed by comparing a representative subset of the universe of low frequency range-based volatility estimators that depend on, and are restricted to, a limited set of historical price observations as their main set of information. Low frequency models do not depend on a higher frequency of intraday price quotes (e.g. tick-by-tick, 1 minute, etc.). Low frequency models depend on the widely available open, high, low and closing (OHLC) prices. Restricting the intraday price observations to the defined low frequency intraday prices ensures that the competing range based volatility estimators meet the parsimony criterion, which is of crucial importance when high frequency intraday price observations are not available. The parsimony criterion includes intraday price observations that are widely available for a vast range of securities and across a wide range of financial markets. This will not be the case for price observations at the very high frequency (e.g. tick-by-tick, 1 minute, etc.) as many securities are not recorded at the very high frequency and many markets lack in trading liquidity to ensure (a sufficient number of) intraday trades at the very high frequency. A reasonable alternative would be to consider a range-based volatility estimator that utilizes only the OHLC price observations, which are widely available for a wide range of securities and markets. The remaining question is the choice of the range based volatility estimator. The total set of competing range-based volatility estimators consists of 13 models of which 7 models exclude overnight returns and 6 models include overnight returns. Overnight returns contain valuable price information when trading activities occur outside of the official trading hours. The overnight change in price can be observed as the difference between the previous day's closing price and the next day's opening price. It is therefore reasonable to expect a positive pay off by including the overnight price information in the volatility estimates.

Following the existing theory on volatility estimation we find that an unbiased benchmark volatility model is based on a set of high frequency intraday price observations. The empirical comparison is performed with different ranking methodologies. The theory on ranking volatility estimates is not consistent in the methodology. Several different ranking methodologies are utilized in this research to compensate for any bias introduced due to dependence on one single ranking methodology. The loss function approach is a common ranking methodology that gives an overall idea of the best fit. This methodology, however, ignores the direction of the movements of the estimates, which can be of crucial importance for the risk management function. The linear correlation as a product of the linear regression implicitly includes the direction of the movement of the volatility estimates. For example, if the volatility estimates of the competing estimators would increase the sign of the direction would be positive, and negative otherwise. When both volatility estimates, the competing one and the benchmark, move in the same direction the correlation would be positive and the other way around. Neither methodologies include non-linear correlation or focus on extreme events, which are often of major interest to risk nor asset management functions. An important product of Copula functions is the upper tail dependence measure, which estimates the precision of the extreme volatilities and takes the signs of the estimates into account. Copula functions are helpful when non-linear correlation exists and needs to be analysed. The latter is exactly the interest of a wide range of risk and asset management functions.

1.2 Research objectives and hypothesis

Volatility of financial prices can be estimated with high accuracy as the frequency of intraday returns increases, providing the preliminary conditions that the intraday returns are uncorrelated and continuous. In this case the high frequency, model-free, volatility estimators is then assumed to be an unbiased estimators of volatility. Volatility can also be estimated with alternative models that require less intraday price observations and are in general less complex to work with. Range-based volatility models that utilize low frequency intraday price observations can contribute to estimating volatility while depending on only a limited number of intraday price quotes. These models are expected to increase the efficiency of volatility estimates that are based on only a single daily observation (e.g. the squared daily return, standard deviation, etc.) by including more valuable intraday price information. At the same time these parsimonious models are expected to achieve a higher overall efficiency compared to volatility models that incorporate intraday price observations at the very high frequency. The overall efficiency is in this case referred to the effort that is required to estimate statistically acceptable volatility estimates. For example, one doesn't need to gain access to costly and specialized data vendors that provide price observations at the very high frequency. High frequency databases contain an extensive number of observations and require data crunching challenges that can be avoided with volatility estimators that utilize low frequency data. Take for example high frequency data that is observed on a "tick-by-tick" basis. The number of observations would increase rapidly with the number of days required for the estimation and, of course, the number of securities for which a volatility estimate is required. Low-frequency range-based volatility models are, in general, also less complex to work with compared to high frequency volatility models. In most cases it can be easily calculated on an Excel spreadsheet while volatility models that incorporate intraday prices at the very high frequency might require more sophisticated software. The literature on range-based volatility estimators provides a wide range of estimators that can be used for estimating the volatility of financial assets. The choice of the range-based volatility estimator is essential for the precision of the estimated volatility and plays therefore a crucial role in various financial assessments like, for example, in risk management.

The main Hypothesis of this research considers the integrated volatility as the 'true' volatility of stock indices. The integrated volatility is based on a continuous set of intraday price observations, which can be approximated by range-based volatility estimators.

H1: Range-based volatility estimators are appropriate models to estimate the 'true' volatility of stock indices.

The 'true' volatility, also referred to as the Integrated Volatility, is estimated with a high-frequency model-free volatility estimator. This model is considered unbiased and is used as a benchmark in this research. The alternative, range-based, volatility estimators that have a positive dependence with the 'true' volatility estimator can be considered appropriate to estimate the 'true' volatility. A set of range-based volatility estimators are tested against the benchmark volatility estimator.

A wide range of research papers on ranking volatility estimates applies a loss function approach to gauge the overall precision. There are, however, many different loss functions to be found in the literature. Hansen and Lunde (2001), for example, ranked volatility forecasts with a wide range of loss functions and provided proof for a set of robust loss functions. This are the minimum squared error and the Q-like function. Loss functions give, however, a first impression of the ranking process as they focus on the overall precision of the volatility estimates. The linear correlation function as a result of the Mincer Zarnowitz regression provides information on the direction of the volatility estimates. As with the loss function approach, the linear correlation also provides a view on the overall precision of the volatility estimates. Similar to previous research on ranking volatility estimators (Lunde (2005) and Laurent, Rombouts and Violante (2009)) this research considers several loss functions to avoid selection bias. It, however, is not obvious which loss function is more appropriate for the evaluation of models, as discussed by Bollerslev, Engle and Nelson (1994) and Diebold and Lopez (1996). To avoid model selection bias this research proposes alternative methodologies for model selection.

Three auxiliary hypothesis have been defined with the main hypothesis:

H.1.1: Range-based volatility estimators are different from each other.

It is important to show whether the set of range-based volatility estimators are indeed different from each other and in some sense unique. This means that each of the estimators has unique properties, which can be leveraged to their advantage when estimating volatility. Following Floros (2009) descriptive statistics and graphical illustration of the results provide a first set of information that describe the statistical properties of the variables. The statistical properties like volatility clustering, platy-kurtosis and non-stationarity are of particular importance. The results of various ranking methodologies are used to show the differences or similarities between a set of range-based volatility estimators.

H.1.2: The efficiency of classical range-based volatility estimators can be increased by including overnight returns.

Overnight returns provide valuable price information outside of normal trading hours. Range-based volatility estimators that don't already include overnight returns in their model can be enriched with the available overnight price information. The research extends a set of classical range-based volatility estimators with overnight return data that proves to include important price information of the stock markets. The performance of these so called 'extended' range-based volatility estimators are compared versus other range-based volatility estimators. The efficiency is determined based on the results of the ranking methodologies.

H.1.3.: Range-based volatility estimators are less biased compared to the squared daily return or the standard deviation.

The squared daily return and the standard deviation are one of the most popular volatility estimators that use (multiple) single daily observations to estimate the volatility. The distance between the estimates and the 'true' volatility estimator can be used for ranking against the range-based volatility estimators. The auxiliary hypothesis states that none of the range-based volatility estimators performs worse than the daily squared return or the standard deviation and thus suggests to use any of the range-based volatility estimators instead of the daily squared return or the standard deviation.

H2: The dependence between the 'true' volatility and range-based volatility estimators is non-linear and shows positive dependence in the tails of the distributions.

An overall ranking methodology might show a good coverage between a range-based volatility estimator and the 'true' volatility, while the coverage during moments of high volatility might be poor. The distribution of high volatility will be in the end of the tail, while the bulk of the volatility will show a much lower scale of volatility. A positive tail dependence between the range-based volatility estimator and the 'true' volatility provides evidence of the performance of the range-based volatility estimator during periods of high volatility. To proof that a range-based volatility estimator performs good during periods of high volatility it is necessary to use a ranking methodology that can rank estimators for their performance in the (extreme) tail of the distribution. For estimating this hypothesis a new ranking methodology based on a Copula function approach is proposed. We argue that for a wide range of risk management functions the ability to include the extremes may be of higher interest then the overall precision of the volatility estimates. After all, extreme losses, bankruptcies and crises are driven by extreme events that are usually not captured. Loss functions could, for example, include a threshold to focus on the extreme movements. However, this approach requires the researcher to define the threshold and is therefore sensitive to the choice of the threshold. This research proposes a novel approach for ranking volatility estimates by applying an upper tail dependence measure to estimate the precision in the extremes. The upper tail dependence is a product of the copula approach that is used as a ranking quantity. The advantage of this approach is that there is no requirement for defining thresholds for when loss functions are used to measure the precision of the extremes.

The primary research objective of this thesis is to find the most efficient low-frequency range-based volatility models that utilize the open, high, low and closing intraday price observations for a respective set of East European stock indices.

The secondary research objective is to show how these results can be applied in practice with a Value-at-Risk application using the .CRBX as an example and a portfolio optimization consisting of several stocks of the .CRBX index.

1.3 Applied scientific research methodology

The significance of volatility modelling can be found in a wide range of research, risk and investment functions. A vast amount of approaches to modelling volatility has been employed in the recent literature. These approaches can be divided in models that consider volatility as an unobservable or an observable variable. The first group includes theories like GARCH and stochastic volatility where volatility is calculated with a parametric model, which requires assumptions about the distribution of the underlying asset. The downside of this approach is that it cannot replicate the main empirical features of financial data and that the estimation procedure may become complex. The second group contains theories which assume that volatility can be observed and relies on a nonparametric approach. This group includes the theory of realized volatility and range-based volatility models, which exploits the available intraday price observations to develop an estimator for the ex-post volatility. A major advantage of realized and range-based volatility models is that it replicates the main empirical features of financial data using realized changes in asset returns.

An important result of properly using the available intraday data is that it can result in unbiased estimators of volatility, i.e. the benchmark. The downturn is that there are several reasons why using high frequency data, e.g. tick-by-tick data, for estimating volatility can introduce challenges. First, existence of microstructure noise can significantly bias the estimator upward as discussed in Andersen et al. (2001) and Alizadeh, Brandt and Diebold (2002). Although this problem can be solved to a large extent by choosing a more advanced estimator for realized volatility it requires some modelling experience. See for example Ait-Sahalia, Mykland and Zhang (2005) who solve this problem by using a Two Time Scale Estimator (TTSE). Second, estimates of realized volatility are driven by intraday trading activities and are therefore unlikely to show intertemporal stability. Barndorff-Nielsen et al. (2008) show that realized volatility estimates may vary substantially on a daily basis. Third, in some asset classes and/or markets there is no intraday data available or the volume of high frequency trades is very low which can impede the estimation of volatility based on high frequency intraday data. Fourth, estimating realized volatility with high frequency data includes working with extensive data sets which may require leading-edge data crunching challenges. While this task is not strictly impossible considering recent developments in the field of information-technology, it remains reasonable to take the cost-effectiveness of this additional effort into account.

To the best of the authors' knowledge, previous research has not covered empirical research of realized volatility and (low-frequency) range-based volatility estimators on the specific European emerging market indices in the presented extent. This is of major importance to financial practitioners in these specific markets as it provides guidelines to the choice of the most efficient volatility estimator in terms of least biased and practical usage. It also applies these estimates in applicable examples of risk and portfolio optimisation studies. This study also enriches the existing literature on volatility estimators by including overnight returns to the respective set of low-frequency range-based volatility estimators.

The literature on ranking methodology has been in an increased focus during the last decennia as it is of crucial importance in determining the least biased volatility estimator. This study enriches the existing literature by utilizing the tail correlation, which is a product of a copula function approach, in the ranking process of volatility estimates.

1.4 Structure of the thesis

The Thesis is structured as follows. Section 2 discusses the high frequency data that is used throughout the research. Secondly it discusses the filtering techniques that are used to filter out different forms of data contamination and, finally, it provides essential statistical analysis of the data.

A theoretical background of the volatility models is provided in the third section. The theoretical background first discusses the theory of realized volatility, which forms the basis for further analysis. Secondly a vast amount of range-based volatility models is described, where the standard deviation, as the most popular volatility estimator both in the literature as well as in practice, is described separately. A vast amount of range-based volatility models that only utilize on open, high low and closing data are described in section 3.2.2. The Realized, a range-based volatility model that utilizes high frequency data, is described in section 3.2.3. Finally, the Two Time Scale Estimator is adopted as a theoretical unbiased estimator of volatility and is applied as a benchmark throughout the research.

Section four discusses the challenges in estimating Realized Volatility and provides alternative methods. It first discusses data challenges when dealing with high frequency data and then turns to the stylized facts of volatility estimation. Thirdly, sampling frequency selection is discussed as sampling frequency has been a popular method to deal with contamination of high frequency data. Finally, conclusions are drawn upon the theoretical analysis provided in this section. Section five discusses an extension of the set of range-based volatility models, where the impact of overnight returns is analysed as it may include important information that is revealed outside of normal trading hours.

The results of the volatility estimates are ranked according to a ranking criteria. The theory of ranking volatility estimates is further discussed in section six. This section discusses several ranking methodologies. The first ranking methodology is based on the efficiency coefficient, which has been a popular methodology of ranking range-based volatility estimates based on low frequency data. Secondly, the Mincer Zarnowitz regression is discussed in section 6.2. Rankings based on efficiency gains, e.g. the mean square error and the quasi likelihood approaches, are discussed in section 6.3. Section 6.4 describes the linear correlation approach. Finally, section 6.5 proposes to use a Copula function approach for when the focus is on the risk in the tails of the distribution rather than on an "overall" best fit.

Sections 8 provides an application in value-at-risk and section 9 in portfolio optimization, where a framework for Realized Covariance matrix based on low frequency price observations is provided. This is an important model in portfolio optimisation.

Finally, section 10 concludes. In the end of the Thesis three appendices have been added to provide insight in the distributions of the realized and range-based volatility estimates and the results of the Mincer Zarnowitz regression.

2 DATA DESCRIPTION

The empirical analysis utilizes a historical set of stock market prices that are observed at a high intraday frequency. This chapter explains the data filtering technique that is applied to the historical dataset of intraday price observations for each of the stock markets. It presents a statistical description of the dataset at different sampling frequencies and provides a graphical representation of the historical price changes for each of the stock markets together with the linear correlation between the different stock markets based on a one day interval.

2.1 High Frequency Data

The data considered in this research consists of the main stock market indices of a set of East European countries with EU membership at the time of writing. The intraday stock market price observations were provided by Thomson Reuters Services and contain the main stock market indices of the following set of Countries: Bulgaria, Romania, Croatia, Czech Republic, Hungary and Poland. The intraday stock market prices are observed at the one minute frequency. The basic information included in the dataset includes the ticker of the index, the date of each transaction, the type denoting "*intraday 1 minute*", the time of the transaction expressed in minutes accurately, and the observed transactional volume. All prices, as provided by Tomson Reuters Services, are denoted in local currency. The data starts on the 4th of January 2010 and ends on the 1st of April 2016. The total number of price observations differ per index due to difference in trading hours, national holidays and overall trading activities. For the purpose of this research it is not necessary to align the dates of the different stock market price observations and neither it is necessary to convert the observed prices to a single currency as the comparison is performed per stock market index.

Table 2-1 gives an overview of the stock indices, tickers, trading dates, trading time and number of observations for each observed index. The start and end date are identical for all stock market indices. The starting and ending time denote the official trading hours for each of the stock market indices. The trading times depend on the internal rules set by the specific stock market. The sample size provides a first glance on the trading liquidity of each of the stock markets. The sample size ranges between 312.644 (.CRBX) and 810.431 (.WIG) observations based on the 1-minute frequency.

Table 2-1 Statistical description of the Index data.

Index		Ticker	Trading Dates		Trading Time		Sample size
Country	City	Ticker	Start	End	Start (time)	End (time)	
Romania	Buchurest	.BETI	4.1.2010	1.4.2016	9:45 AM	6:00 PM	327.145
Hungary	Budapest	.BUX	4.1.2010	1.4.2016	9:00 AM	5:10 PM	639.294
Croatia	Zagreb	.CRBX	4.1.2010	1.4.2016	9:15 AM	4:25 PM	312.644
Chech Republic	Praha	.PX	4.1.2010	1.4.2016	9:00 AM	4:25 PM	630.858
Bulgaria	Sofia	.SOFIX	4.1.2010	1.4.2016	9:30 AM	5:15 PM	571.885
Poland	Warsawa	.WIG20	4.1.2010	1.4.2016	9:00 AM	6:00 PM	810.431

2.2 Filtering technique

Pricing quotes sampled at ultra-high velocity are prone to wrong data records either due to the asynchronous nature of tick data, the treatment of time, differences in tick frequencies, etc. If these type of errors exist in the recorded database then they need to be dealt with accordingly. For analytical purposes the data also needs to be presented in discrete and equidistant or equal time intervals. As a consequence tick-by-tick data are subject to sophisticated filtering and data cleaning algorithmic techniques as proposed by, e.g., Brownlees and Gallo (2006) and Barndorff-Nielsen et al. (2006).

The database received from Reuters is already presented at a discrete time sampling intervals of 1 minute and is therefore not prone to the type of errors that contaminate ultra-high frequency data as for example would be the case with tick-by-tick data. However, the type of errors that may contaminate this type of data are mostly driven by missing intraday quotes, e.g. due to lack of trading activities in particular markets, and price observations outside of normal trading hours. Using ultra-high sampling frequencies in these markets like for example tick-by-tick or even 1 second sampling would result in predominantly empty fields for most of the indices due to limited trading activities that is determined by the market. The choice for the high frequency sampling interval of 1 minute is based on the least complete index, i.e. the .CRBX. For the reasons outlined we suggest to avoid ultra-high sampling frequencies when dealing with this type of markets and propose a fairly simple filtering technique to ensure that only non-zero quotes within official trading hours are included. Our filtering technique consists of a two-step approach. In the first step only transactional prices observed during official opening hours of the particular index are included in the analysis. All quotes outside of official opening hours are removed from the dataset. In this way the filtering technique ensures that only official price quotes are included in the analysis. In the second step all price observations denoting zero are removed from the dataset as it are considered recording errors. No other outliers have been detected in the dataset.

2.3 Statistical properties of the data set

The dataset consists of a set of consecutive and discrete intraday price observations sampled at a 1 minute frequency. Figure 2-1 shows a sample of 170 intraday transactional prices for .CRBX that have been observed during the 18^{th} of January 2016. The beginning of the trading day starts at 9:15AM and ends at 4:25PM. Even at the 1-minute frequency Figure 2-1 shows that there are empty spots, which indicate that there hasn't been any trading activity in this index during some periods. Overall the 1-minute frequency shows that in most 1-minute intervals a trading activity has occurred.

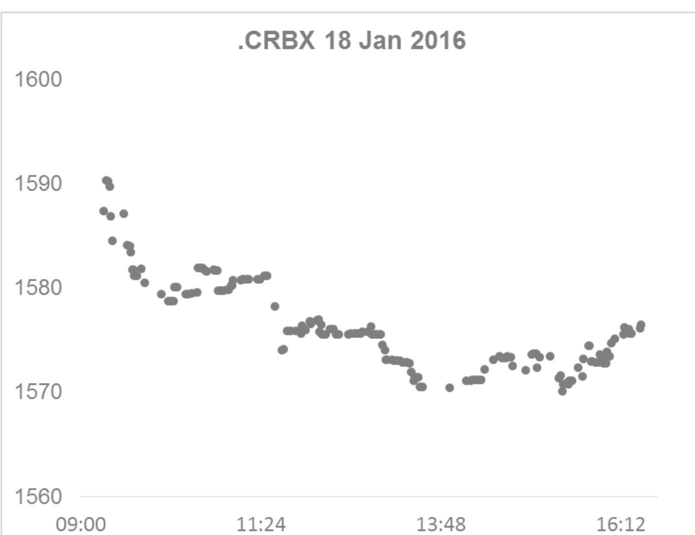


Figure 2-1 Sample of intraday transaction prices of the .CRBX index during 18 January 2016.

A sample of a set of consecutive transactional intraday price movements indicating a trading week is shown in Figure 2-2. The vertical bold lines indicate the ending and beginning of a trading day. Notice that the standard deviation, which is a popular volatility estimator, would usually only include the prices observed at the end of the day. Hence, the standard deviation ignores all the intraday price information that is available. Notice that on the 22nd of January the lowest price observation was lower than the opening or closing price and that on the 23rd of January the highest price observation exceeded both the opening as well as the closing price. The difference between the closing price on the 23rd of January and the opening price on the 24th of January is defined as an "overnight jump". Overnight price jumps indicate the price change between the ending and beginning of a new trading day and often contains valuable price information.

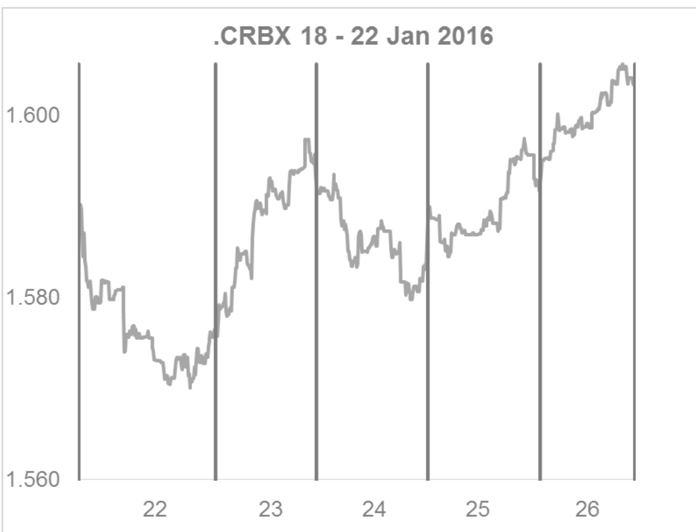


Figure 2-2 Sample of intraday transaction prices of the .CRBX index during the period of 22-26 January 2016. The vertical lines indicate the beginning and the end of each trading day.

The entire database for all the stock market indices is visualized in figure 2-3. This figure shows the price range during the observation period and the price path for each index. From figure 2-3 we observe that the price movements between .BUX and .BETI show some similarities during the observation period. To some extent a similarities between price movements can be observed with .CRBX, .WIG20 and .PX.

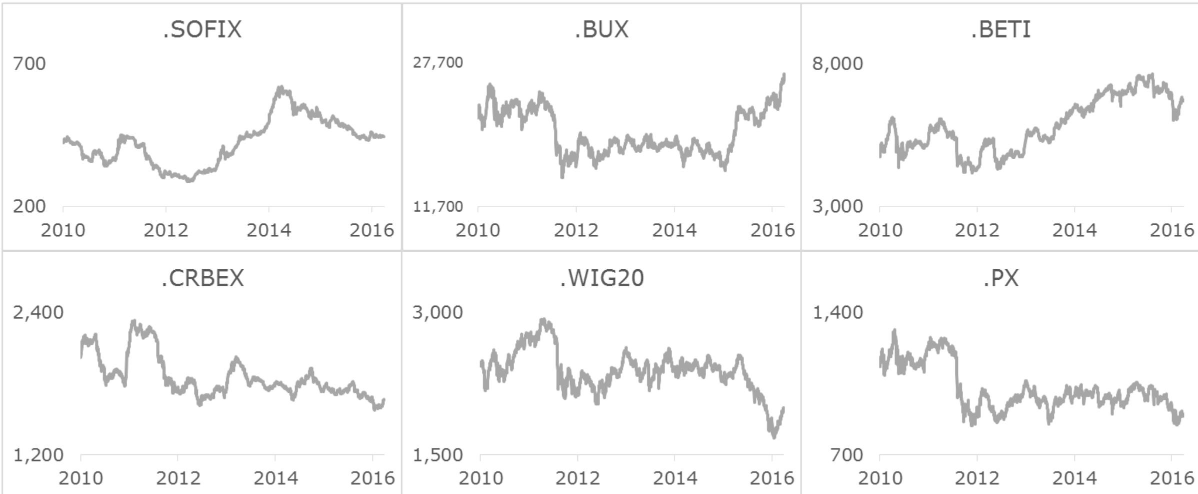


Figure 2-3 Historical price observations of selected indices.

Table 2-2 shows the linear correlations between the selected indices. All correlations greater than 50% are shown in bold. From the correlation matrix we observe that .BETI shows strong correlation with .CRBX, .PX and .BUX, while .CRBX is also strongly correlated with .BUX. Only .SOFIX and .WIG20 show poor correlation with all other indices except between each other.

Table 2-2 Linear correlations of the selected indices.

Correlation	.SOFIX	.CRBEX	.PX	.BUX	.WIG20	.BETI
.SOFIX	1.00	-0.06	0.00	0.07	0.80	0.01
.CRBEX	-0.06	1.00	0.38	0.68	-0.21	0.81
.PX	0.00	0.38	1.00	0.13	0.18	0.62
.BUX	0.07	0.68	0.13	1.00	-0.11	0.73
.WIG20	0.80	-0.21	0.18	-0.11	1.00	-0.07
.BETI	0.01	0.81	0.62	0.73	-0.07	1.00

The database with the sampling frequency of 1 minute allows us to sample the data at lower frequencies, e.g. 5 minute, hourly, etc. The reason for using intraday price observations is to include this information in the volatility estimator. However, in the existing literature there is no consensus reached on the optimal sampling frequency for estimating volatility. See for example Zhang et al. (2005), Hansen and Lunde (2006) and Bandi et al. (2008) who propose different optimal sampling frequencies for an optimal volatility estimate. A conclusion that can be drawn from the different analysis made is that the optimal sampling frequency depends on the type of security and its intraday trading activity. A limit on the highest possible frequency can be suggested by means of intraday trading activities. For example, the observed markets show very poor intraday trading activities at the 1 second interval, while at the 1 minute interval the trading activity becomes substantial. Another aspect to consider is the extent to which the optimal sampling frequency is time invariant, i.e. the optimal sampling frequency might change over time due to mutations in the nature of the underlying security. This research takes the type of market into account when determining the optimal sampling frequency while the time period of determining the optimal sampling frequency is left for further research in this area. Table 2-3 shows the basic statistics for the entire dataset given the highest possible sampling frequency and the database at disposal. Next to the 1 minute frequency the statistics of 5 minute, hourly and daily frequencies are assessed. The mean, median and standard deviation increase with the sampling frequency, which means that on average the price changes increase with the frequency.

Table 2-3 Descriptive statistics of the stock index returns during the period 4.1.2010-1.4.2016.

Index	Sampling frequency	Mean	Median	Standard deviation	Skewness	Kurtosis	Sample Size
Romania	1 minute	1,1E-06	0,0E+00	7,6E-04	-3,08	579,62	327,145
	5 minute	4,3E-06	0,0E+00	1,5E-03	-1,77	157,09	327,142
	Hourly	2,8E-05	9,7E-06	3,9E-03	-0,67	28,92	327,119
	Daily	1,5E-04	2,4E-04	9,4E-03	-0,87	17,73	327,006
Hungary	1 minute	3,4E-07	0,0E+00	6,5E-04	-0,24	331,35	639,294
	5 minute	4,0E-07	0,0E+00	1,4E-03	-0,38	88,23	571,881
	Hourly	4,0E-06	0,0E+00	4,8E-03	-0,13	10,02	571,828
	Daily	3,3E-05	3,3E-04	1,3E-02	-0,40	2,90	571,464
Croatia	1 minute	-5,8E-07	0,0E+00	4,8E-04	12,01	1828,77	312,644
	5 minute	-2,9E-06	-5,6E-06	1,1E-03	5,21	375,35	312,640
	Hourly	-2,4E-05	-7,6E-05	3,3E-03	1,19	52,06	312,602
	Daily	-1,5E-04	-1,3E-04	8,2E-03	0,59	11,42	312,380
Chech Republic	1 minute	-3,6E-07	0,0E+00	4,7E-04	-1,01	1460,01	630,858
	5 minute	-1,8E-06	0,0E+00	1,1E-03	-0,60	213,80	630,854
	Hourly	-1,8E-05	0,0E+00	3,5E-03	-0,42	21,48	630,810
	Daily	-1,5E-04	3,4E-04	1,1E-02	-0,63	6,77	630,430
Bulgaria	1 minute	7,6E-08	0,0E+00	4,7E-04	-0,50	209,90	571,885
	5 minute	4,2E-07	0,0E+00	1,1E-03	-0,68	68,40	571,881
	Hourly	2,8E-06	0,0E+00	2,7E-03	-0,21	28,35	571,856
	Daily	2,3E-05	0,0E+00	8,0E-03	0,04	10,79	571,616
Poland	1 minute	-2,5E-07	0,0E+00	5,2E-04	-2,69	453,49	810,431
	5 minute	-1,6E-08	0,0E+00	1,2E-03	-0,82	81,85	571,881
	Hourly	1,5E-08	0,0E+00	4,0E-03	-0,37	10,29	571,828
	Daily	-3,6E-06	2,7E-04	1,2E-02	-0,57	4,55	571,432

3 THEORY OF VOLATILITY ANALYSIS

The first section of this chapter discusses the theory of realized volatility, which is the basis of this research. The second section discusses the theory of range based volatility estimators, which are the alternative estimators of the realized volatility and will be further assessed in an empirical analysis. The third section discusses an important question related to sampling frequency selection and unbiased estimators of the true integrated volatility. The Two Time Scales approach is proposed to avoid bias due to microstructure noise or sampling error. This approach is assumed to provide with an unbiased estimator of the true (integrated) volatility and is adopted as the benchmark volatility estimator in this research.

3.1 The Theory of Realized Volatility

The actual volatility, also referred to as the integrated volatility (IV), is a measure of the ex-post return variability over a defined and non-vanishing time interval. This is a theoretical framework that provides the basis for formulating a discrete model that can be used to quantify the actual volatility given a certain set of assumptions. The theoretical framework starts with an assumption of the underlying data generating process.

Let r_t denote the logarithmic return of the instantaneous stock price p_t of a particular security at day *t*, hence $r_t = \log\left(\frac{p_t}{p_{t-1}}\right)$.

Assume that the underlying data generating process is a continuous time, continuous sample-path model, and that the logarithmic returns follow a Brownian semi-martingale of the form

$$r_t^h = \int_t^{t+h} \mu(u)du + \int_t^{t+h} \sigma(u)dWu \qquad \text{3-1}$$

The first term of the process on the right hand side is the drift, where μ is a local martingale with constant variable and finite variation. The second term on the right hand side is the diffusion coefficient, where σ^2 is a càdlàg adapted stochastic volatility process of locally bounded variation away from zero and dWu is a standard Brownian motion. The volatility σ^2 is also independent of Wu. The integrated volatility is defined as the integral of the instantaneous volatility over the one day interval, $[t, t+h]$, where the time interval h represents a full 24 hours day. The IV can be considered an unbiased latent volatility measure and is of the form:

$$IV_t^{(h)} = \int_t^{t+h} \sigma^2(u)du \qquad 3\text{-}2$$

In practice, however, stock price observations are observed in discrete time intervals and are usually not continues. The analysis of the data in section 2.3 shows the sampling discreteness of the East European indices. The data are observed with a frequency of 1 minute and have both an opening and a closing time during the day. There are no price changes observed prior to the opening nor after the closing of any of the indices. To estimate the actual volatility a discrete formulation of the IV is required. As noted in Barndorff-Nielsen and Shephard (2001), Andersen, Bollerslev, Diebold and Labys (2001) and also in the earlier work of Comte and Renault (1998) the IV of the semi-martingale process in equation 3-1 can be estimated using cumulative squared intraday returns observed with high frequency. Hence the standard definition of realized volatility (RV) of returns is defined as the sum of intraday squared returns, i.e.

$$RV_{t,\Delta} \equiv r_{t,\Delta}^2 = \sum_{j=1}^{M} r_{t,j,\Delta}^2 \qquad 3\text{-}3$$

Assuming that the interval between observations is equally distanced, commonly referred to as equidistant, a total of M intraday returns can be constructed from the opening to the closing time of the indices. The sum of squared intraday returns is an unbiased measure of the integrated volatility under some general conditions. The theoretical justification for this approach is that when the number of observations, M, goes to infinite the RV tends in probabilistic terms to the quadratic variation of the semi-martingale process (equation 3-1). It is therefore a consistent, unbiased and nonparametric estimator of the IV over the fixed time interval, i.e. $plim_{m \longrightarrow \infty} RV_{t,\Delta} = \sigma_{t,\Delta}^2$. However the intraday returns must be continuous serially uncorrelated and there should exist no bid-ask bounce or any form of microstructure noise that contaminates the result. Although these condition perhaps do not seem unreasonable, in practice the assumption of continuous returns is often violated because securities usually have limited trading hours. There is one exception in the foreign currency market which is traded for almost 24 hours a day and seven days a week. The assumption of serially uncorrelated returns is also often violated and can be seen as one of the stylized facts of high frequency financial return time series. Thus in practice the RV is very seldom an unbiased estimator of the latent true volatility (Zhang et al. (2005), Bandi et al. (2008)).

Figure 3-1 shows the estimated RV for the observed set of East European stock indices.

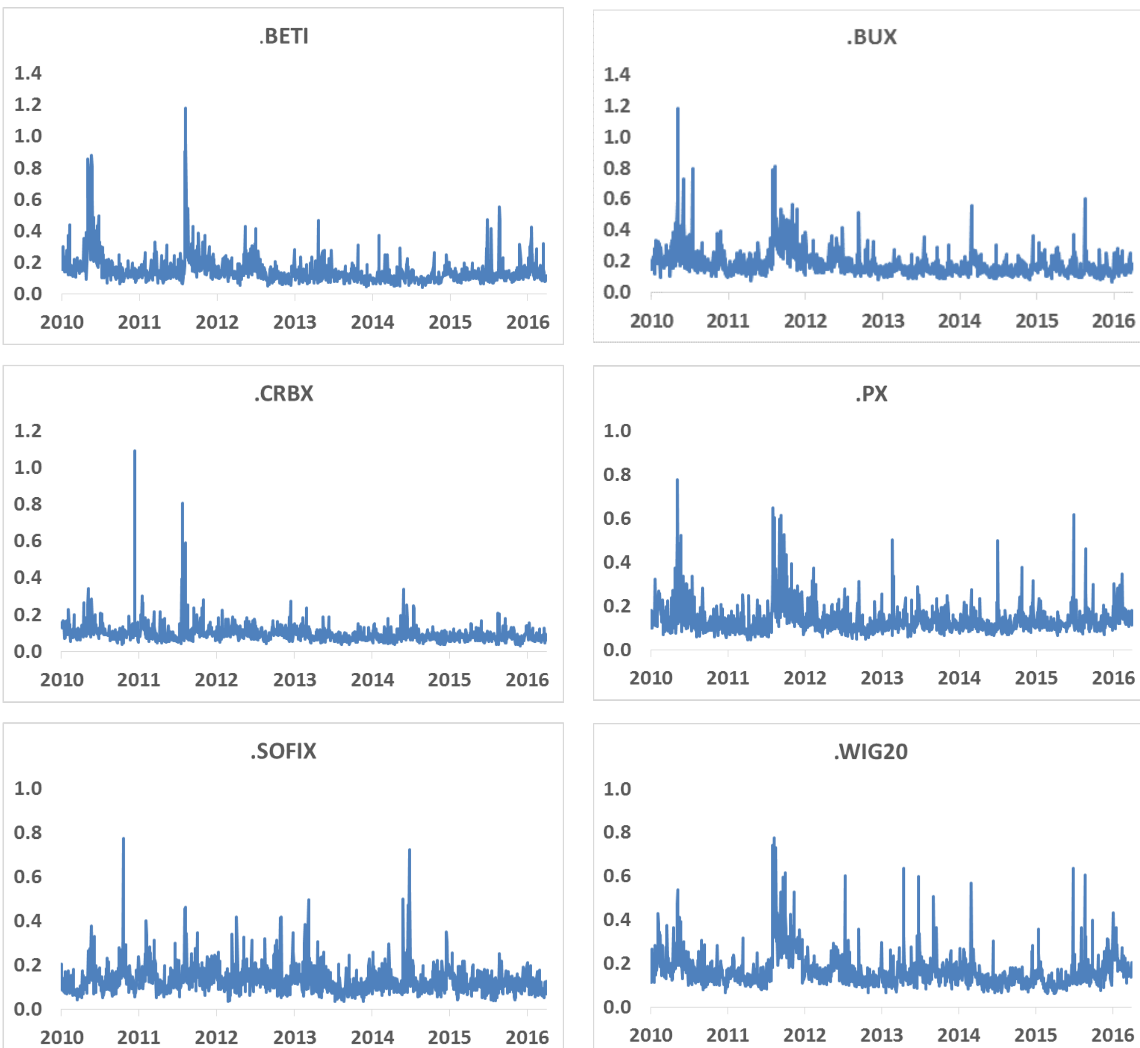


Figure 3-1 shows the Realized Volatility estimates of the observed East European indices during

2010-2016.

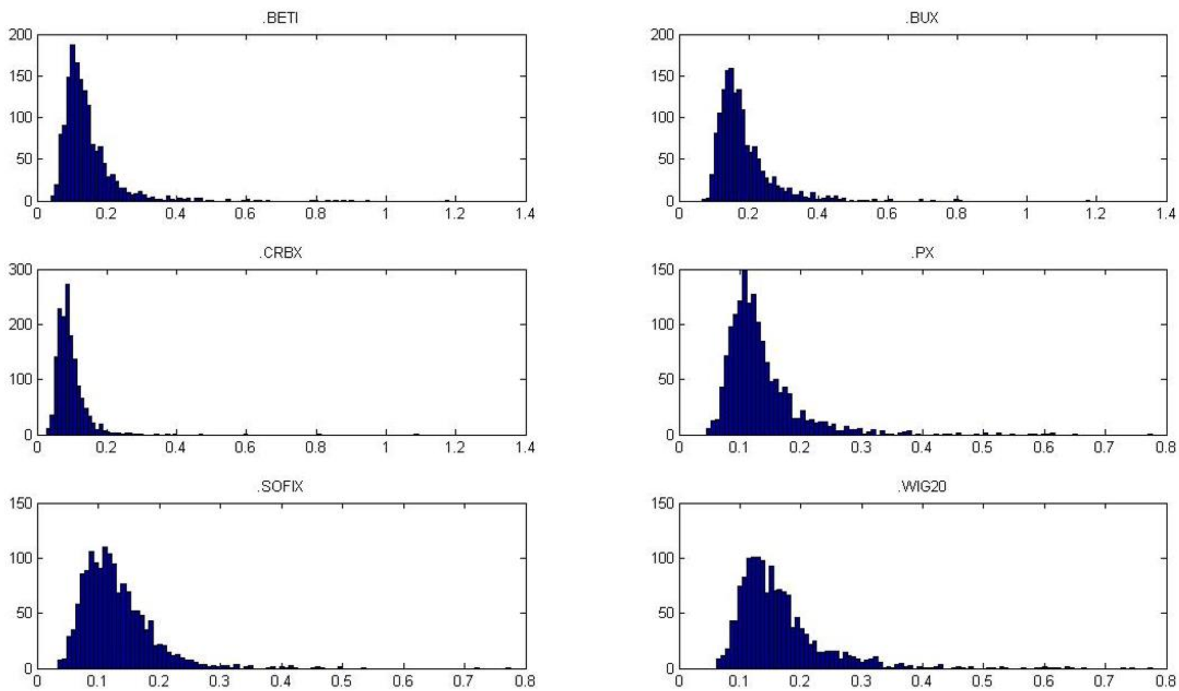


Figure 3-2 Historical distributions for six East European indices.

Figure 3-2 shows the distributions of the RV estimates for the 6 East European indices. It is evident that .BETI, .BUX and .CRBX show heavier tails compared to .PX, .SOFIX and .WIG20. Table 3-1 shows the statistical properties of realized volatility estimates for the 6 stock market indices. The average of the daily returns ranges between 0.095 (.CRBX) and 0.186 (.BUX). The standard deviation ranges between 0.052 (.CRBX) and 0.091 (.BETI). The positive skewness indicates that the distribution is skewed to the right side, which is also evident from Figure 3-2. The sample sizes of the indices ranges between 1546 (.SOFIX) and 1577 (.BETI) and can be considered reasonable in line across all indices.

Table 3-1 Statistical properties of realized volatility estimates for the European indices.

Statistics	**.BETI**	**.BUX**	**.CRBX**	**.PX**	**.SOFIX**	**.WIG20**
Mean	0.146	0.186	0.095	0.137	0.132	0.169
Median	0.125	0.165	0.086	0.120	0.120	0.149
Standard deviation	0.091	0.085	0.052	0.070	0.065	0.082
Skewness	4.413	3.400	8.036	3.349	2.799	2.815
Kurtosis	30.527	21.589	121.991	17.272	15.855	11.954
Sample Size	1577	1554	1557	1568	1546	1562

3.2 The Theory of Range based Volatility estimators

Range-based volatility estimators utilize the available intraday price information by combining the spread between the open, high, low and closing prices. The range can be defined within any subset of intraday price observations based on any available frequency. The easiest example can be found with the daily frequency, where the opening price equals the first observable price at the opening of the market, the high and low indicate the extremes during the trading day, while the closing price indicates the final closing price at the end of the trading day. When dealing with high frequency data the high and low are defined as the extremes subset within a trading day. Range-based volatility estimators can depend on high frequency intraday data (see for instance section 3.2.3, where the realized range is discussed in more details) as well as on low frequency intraday data. Estimators that require only a limited amount of intraday observations consisting of a combination of the open, high, low and closing prices belong to the set of low-frequency volatility estimators. Low frequency data has some practical advantage of being widely available across a vast amount of asset classes and markets. Besides this, volatility estimators based on low frequency data have in general a computational advantage by being less complex compared to estimators that require high-frequency data. One of the most popular low frequency volatility estimators that can be found in the literature is the standard deviation, which is discussed in more details in section 3.2.1. Section 3.2.2 discusses a wide range of range based volatility models based on low frequency OHLC data that can be found in the literature. Finally, section 3.2.3 discusses the Realized Range estimator that utilizes high frequency price observations.

3.2.1 Standard deviation

The standard deviation of financial returns is a volatility estimator which shows how the data is clustered around its mean. For statistical analysis it is an indispensable statistic as other statistics, like the skewness and correlation for example, also depend on the standard deviation. Although there is no officially market standard model for estimating the market volatility, it wouldn’t be a completely false statement to assert that the standard deviation of returns is the most widespread used model for estimating market volatility. This would then be the case in both the literature as well as in practice. For example, it is widely used in Modern Portfolio

Theory, which was introduced by Nobel Laureate Harry Markowitz (1952) in his seminal paper which changed the way portfolios were managed until then. It is also widely used in the theory of Value-at-Risk (VaR) of which Philippe Jorion (2007) can be considered as one of the 'fathers' of the modern VaR theory. The standard deviation was also one of the key assumptions in the Black and Scholes (1973) option pricing model.

The standard deviation owes this popularity to, among others, its relatively simple calculation on one side and the intuitive symmetric assumption that financial returns follow a normal distribution on the other side. It is a metric of the volatility that can be considered in relationship to the mean of the distribution of returns. This assumption, however, is often violated because the distribution of financial returns is often not symmetric, but rather 'suffers' from positive skewness and leptokurtosis which are one of the well-known 'stylized facts' of financial time series. Ignoring these stylized facts can result in miscalculated volatility estimates, which may undesirably influence all further thereon dependent decisions. Another aspect of concern is that the standard deviation requires multiple consecutive daily price observation and thereby ignores the intraday price movements and all information that is available during the day. For example, assume an extreme situation where the intraday price movements in several consecutive days are extremely volatile, yet close each day on the same level. The standard deviation based on closing prices would in this case detect zero volatility, while the estimated volatility would have been positive if the intraday mutations were considered. To correctly estimate the volatility of financial returns it is important to take the intraday price movements into account.

Another drawback of the standard deviation is that it depends on a sequence of historical observations, i.e. it is not possible to estimate the daily volatility based on the information observed during a particular day. The standard deviation heavily depends on the number of historical observations that are required for the estimation. This sensitivity raises the classic question on the optimal number of observation for estimating the standard deviation.

Figure 3-3 shows the impact of the standard deviation for 6 stock indices based on 10 and 100 day historical observations.

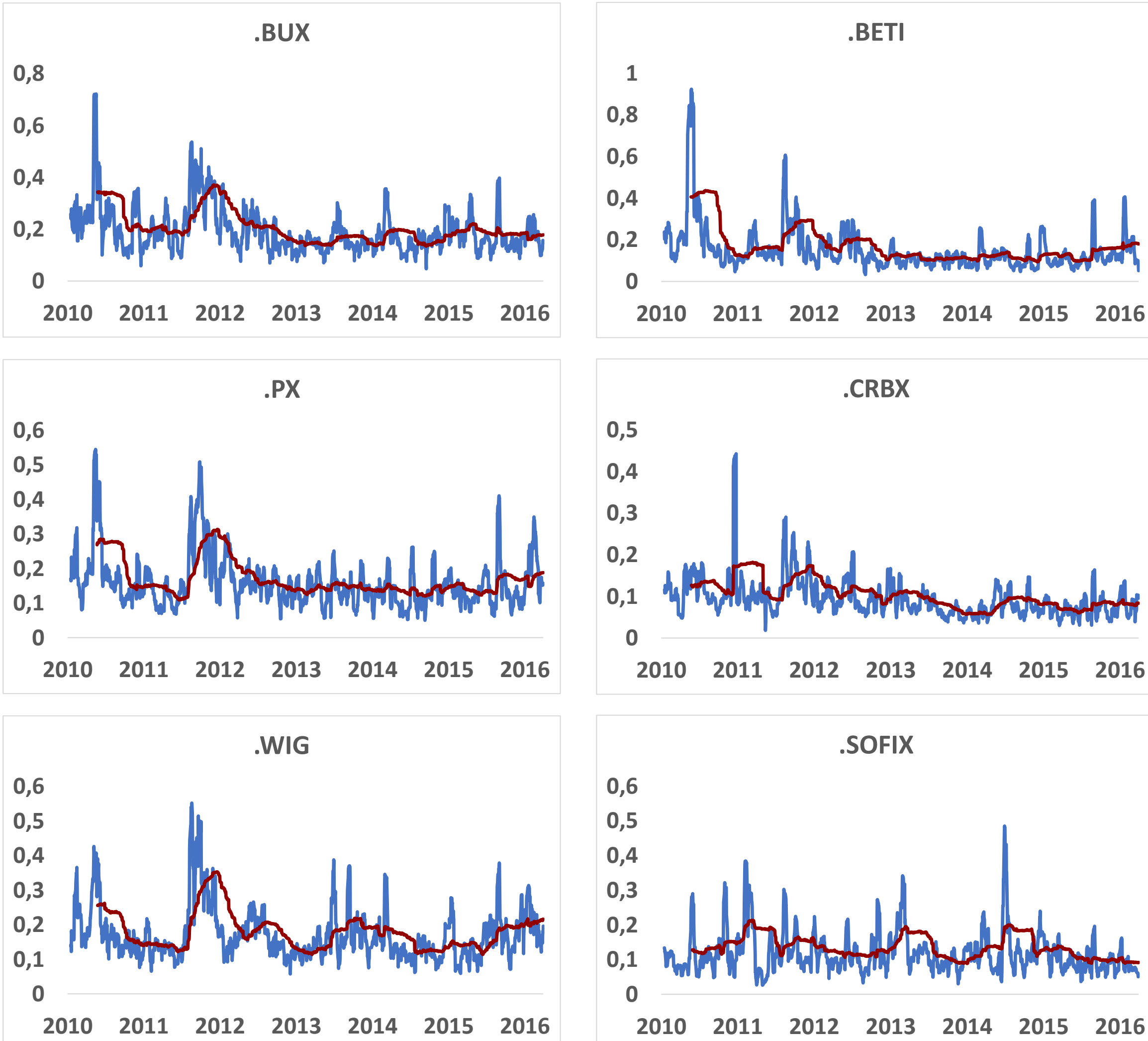


Figure 3-3 The annualized historical standard deviations based on 10 and 100 observations denoted, respectively, with the blue and red line for the period 2010-2016.

It is obvious that the standard deviation based on 100 trading days, i.e. the red line, is less volatile than the standard deviation based on 10 trading days, i.e. blue line. Hence the more trading days are used to estimate volatility the smoother the estimator becomes.

3.2.2 Range based volatility estimators

Range based volatility estimators that utilize the information available from OHLC type of data, generally assume that asset prices follow a Geometric Brownian Motion (GBM). This means that the price of the asset on day *t* is independent of the price of the same asset on day *t-1* and that the price of the assets are stochastic through time. A vast number of range based volatility models that utilize OHLC type of data have emerged in the literature during the past few decennia. This section provides a literary overview of the range based volatility models.

First we explain the notations of the volatility estimators. Let's denote the intraday price of a financial asset on day $i \in \ I = \{0,1,2,...\}$ at time $\tau \le t \le T$ as $P_{t,i}$. Denote the daily closing price on day i a C_i , the daily opening price as O_i , the daily lowest price as L_i and the daily highest price as H_i . Define a set of range-based volatility estimators as $\hat{\sigma}^2_{t,i}$, where $i = 1,2,..n$ denotes the number of estimators and *t* denotes the time in days. Hence, the range-based volatility estimators use a combination of open, high, low and closing prices to estimate the market volatility. Range-based volatility estimators are therefore subject to a limited sampling frequency of at least one and at most four observations per single trading day. Range-based volatility estimators use a combination OHLC price observations during a single trading day or during multiple consecutive trading days.

The formula for estimating the volatility presented in this chapter assumes that the volatility of a single day is in the focus of interest. However a longer period of observations can also be taken into account as well. The ex-post volatility can be calculated with a number of consecutive historical days. Although the number of days used to estimate volatility is arbitrary, it has a lower limit of 1 day. For ease of comparison the estimated volatilities can be annualized with the number of trading days per annum, *N*, set to 250. The annualized volatility estimate $\hat{\sigma}^*_{t,i}$ at time t is denoted by:

$$\hat{\sigma}^*_{t,i} = \sqrt{\frac{N}{F}} \cdot \sqrt{\sum_{t-F}^{t} \hat{\sigma}^2_{t,i}} \qquad 3\text{-}4$$

One of the most popular and simplest measures that can be used for estimating daily volatility is the daily (close-to-close) squared (D). The return is based on the natural logarithm of closing prices observed on two consecutive days. The formula for the close-to-close estimator based on two closing price observations on two consecutive trading days is of the form:

$$\hat{\sigma}^2_{i,D} = \left(\ln \frac{C_i}{C_{i-1}} \right)^2 \qquad 3\text{-}5$$

The advantage of the close-to-close estimator is that it requires only daily closing price observations. It can be calculated based on only 2 consecutive trading days, but can be extended to closing prices observed in multiple trading days. The disadvantage of the close-to-close estimator is that it doesn’t include intraday price movements. As intraday price movements contain information on the volatility of the financial asset it is considered important. Alternative popular measures of volatility that exploit intraday price range information are the daily close-to-open (CO), the daily close-to-open-to-close (COC) and the daily high-low (HL) volatility measures

$$\hat{\sigma}_{i,CO}^2 = \ln\left(\frac{C_i}{Oi}\right)^2 \quad \text{3-6}$$

$$\hat{\sigma}_{i,COC}^2 = \ln\left(\frac{C_i}{Oi}\right)^2 + \ln\left(\frac{O_i}{C_{i-1}}\right)^2 \quad \text{3-7}$$

$$\hat{\sigma}_{i,HL}^2 = \ln\left(\frac{H_i}{L_i}\right)^2 \quad \text{3-8}$$

The advantage of the close-to-open-to-close estimator is that includes overnight price mutations. The High-Low uses the highest and lowest intraday available price change to estimate the volatility. Since 1980 a number of models for estimating daily volatility have emerged in the literature. The efficiency of these models has been measured with the coefficient of efficiency, which is commonly compared to the daily squared return estimator (equation 3-5) as the usual benchmark in the wide literature. Chapter 7.1 explains the coefficient of efficiency in more detail. In this section we only use the results of the coefficient of efficiency that is found in the literature.

One of the first to propose a range based volatility estimation model that assumes an underlying geometric Brownian motion with zero drift, $\mu = 0$, was Parkinson (1980). This model uses the daily variance measured as the difference between the maximum and the minimum intraday price for estimating the volatility. According to Parkinson (1980) this new volatility measure that is based on extreme high and low intraday price ranges has an efficiency coefficient that is approximately five times higher than the classical daily squared returns.

$$\hat{\sigma}_{i,P}^2 = \frac{1}{4\ln(2)} \cdot \ln\left(\frac{H_i}{L_i}\right)^2 \quad \text{3-9}$$

Garman & Klass (1980) also assume zero drift, but include all four OHLC intraday price observations by incorporating the opening and closing prices in their model. Garman & Klass (1980) claim to gain in the efficiency coefficient approximately 8 times in comparison to the daily squared return

$$\hat{\sigma}^2_{i,GK} = \frac{1}{2}\left(\ln\frac{H_i}{L_i}\right)^2 - (2\ln 2 - 1)\left(\ln\frac{C_i}{O_i}\right)^2 \quad 3\text{-}10$$

One of the main disadvantages of the Parkinson and the Garman & Klass models is the restrictive assumption that has been set on the drift of the assumed price process, i.e. $\mu = 0$. Rogers & Satchell (1991) and Rogers, Satchell & Yoon (1994) propose a drift independent estimator when the drift term for the prices is not zero. This volatility estimator is particularly useful when the drift of the underlying geometric Brownian motion is non zero.

$$\hat{\sigma}^2_{i,RS} = \ln\frac{H_i}{C_i}\ln\frac{H_i}{O_i} + \ln\frac{L_i}{C_i}\ln\frac{L_i}{O_i} \quad 3\text{-}11$$

An extension of this model is proposed by Yang and Zhang (2000). This estimator combines the classical and the Roger and Satchell estimator and also allows for opening jumps.

$$\hat{\sigma}^2_{i,YZ} = \hat{\sigma}^2_{i,O} + k\cdot\hat{\sigma}^2_{i,C} + (1-k)\cdot\hat{\sigma}^2_{i,GK} \quad 3\text{-}12$$

The 'opening' volatility is defined by $\hat{\sigma}^2_{i,O} = \left(\ln\frac{O_i}{C_{i-1}} - \overline{O}\right)^2$, the 'closing' volatility by $\hat{\sigma}^2_{i,C} = \left(\ln\frac{C_i}{O_{i-1}} - \overline{C}\right)^2$ and $\hat{\sigma}^2_{i,GK}$ is the volatility measure as proposed by Garman and Klass (1991). The constant k takes the form $k = \frac{\alpha}{1+\frac{m+1}{m-1}}$. According to Yang and Zhang the optimal value for the parameter α is 0.34. Given the specificities of the underlying financial time series the optimal value for the parameter α can additionally be calibrated by minimizing the mean squared error in comparison to the benchmark unbiased volatility estimates. The Yang and Zhang model can be considered as a more complete model since it allows for opening jumps and it also includes previous day opening and closing prices.

A summary of the various range-based volatility estimates is included in table 3-2. The summary table indicates which of the available (open, high, low and closing) prices are used by the estimator. Next the summary table also indicates whether previous day prices, drift and/or overnight jumps are included. Thus some of the estimators only use current day price observations, while other estimators also rely on previous day price observations.

Table 3-2 Summary of range-based volatility estimates.

Volatility estimate		Prices taken	Include previous day prices	Include Drift	Include o/n jumps	Theoretical efficiency gain
$\hat{\sigma}^2_{i,CCS}$	Standard Deviation	C	Yes	No	No	-
$\hat{\sigma}^2_{i,CCS}$	Close-to-Close squared daily return	C	Yes	No	No	1
$\hat{\sigma}^2_{i,CCA}$	Close-to-Close absolute daily return	C	Yes	No	No	-
$\hat{\sigma}^2_{i,CO}$	Close-to-Open	CO	No	No	No	-
$\hat{\sigma}^2_{i,HL}$	Range	HL	No	No	No	-
$\hat{\sigma}^2_{i,P}$	Parkinson	HL	No	No	No	5.2
$\hat{\sigma}^2_{i,GK}$	Garman-Klass	OHLC	No	No	No	7.4
$\hat{\sigma}^2_{i,RS}$	Roger-Satchell	OHLC	No	Yes	No	8
$\hat{\sigma}^2_{i,YZ}$	Yang-Zhang	OHLC	Yes	Yes	Yes	14

The drift is an important assumption for each estimator. Not assuming a drift term means that the estimator assumes zero drift of the underlying Brownian semi-martingale. Overnight jumps are important when there have been price changes overnight and the current opening price is not equal to the previous closing price. Indicating that the market has reacted to information outside of normal trading hours. In this case it is import for an estimator to include overnight jumps. The theoretical efficiency gain is based on efficiency gains from existing literature. The close-to-close squared daily return has an efficiency gain of 1 by default. The variance of the alternative volatility estimators is compared to the squared daily return. More details on the efficiency gain can be found in Yang and Zhang (2000).

3.2.3 Realized Range

The Realized Range model builds further on the theory of Realized Volatility and incorporates intraday range based information to increase the efficiency of the estimation. The model follows Parkinson (1980) by replacing each squared intraday return of the Realized Volatility estimate by the high-low range for each intraday sub period. Following Martens and van Dijk (2006) and Christensen and Podolskij (2006) the formulation is of the form

$$RR_i = \frac{1}{4\ln 2}\sum_{i=1}^{\tau}\left(\ln H_{t,i} - \ln L_{t,i}\right) \qquad \text{3-13}$$

Where H and L represent the highest and lowest prices of the i^{th} intraday interval on the t^{th} trading day, respectively. Martens and van Dijk (2007) consider a bias adjustment procedure to account for microstructure effects. They propose to scale the realized range by using a ratio of the average level of the daily range and the average level of the (scaled) realized volatility. The scaled, bias adjusted Realized Range is of the form

$$RR_i^* = \left(\frac{\sum_{l=1}^{q} RR_{i-l}^{D}}{\sum_{l=1}^{q} RR_{i-l}}\right) RR_i \qquad \text{3-14}$$

Where RR_i^D denotes the daily range and q the number of trading days. The main idea behind the scaling factor is that the daily range based volatility is free of microstructure noise. An important criteria for the Realized Range is that it requires intraday sampling at very high frequency, to ensure a sufficient number of intraday sub periods to calculate the Realized Range.

3.3 Two time scales approach

The Two time scales estimation model builds further on the theory of Realized Volatility. Market microstructure noise in the RV can be filtered out by using a sub sampling method as proposed in Zhang et al. (2005) and in Ait-Sahalia, Mykland and Zhang (2005). The two time scale estimator (TTSE) estimates the IV consistently in the presence of microstructure noise. The TTSE uses the highest possible sampling frequency, N, to filter out the magnitude of the noise term by subtracting it from the average sparse RV estimator. The sparse RV estimator uses a lower sampling frequency, Δ, to estimate the average RV over a total of S non-overlapping return series. For example, if the sampling frequency is 10 minutes than the first RV uses the returns sampled at 9:00, at 9:10, etc. The second RV is calculated using returns sampled at 9:01, 9:11, etc. and the last RV is calculated by returns sampled at 9:09, 9:19, etc. The average RV using sparse sampling frequencies is

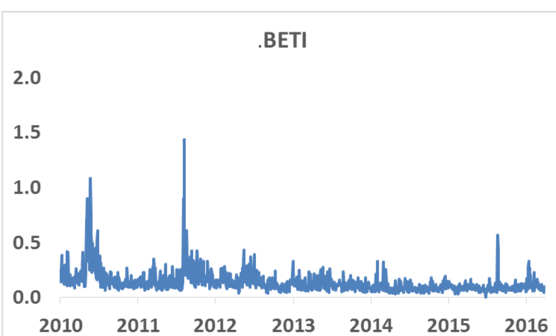
$$RV_t^{\Delta,S} = \frac{1}{S}\sum_{s=1}^{S} RV_{t,s}^{\Delta} \qquad 3\text{-}15$$

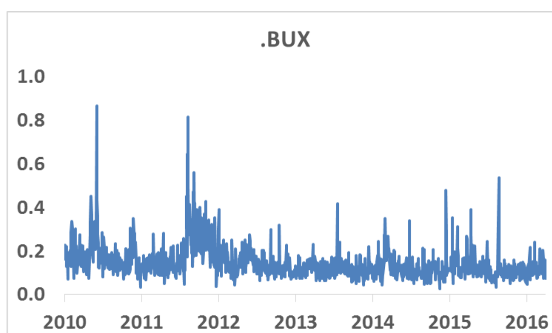
Where S denotes the number of high frequency intraday observations. TTSE is calculated by subtracting the average of the sparse RV by an adjusted RV based on a high frequency price observations. The first term on the right side of equation 3-16 denotes the filtering term.

$$TTSE_t^{\Delta} = RV_t^{\Delta,S} - \frac{\bar{n}}{N} RV^N \qquad 3\text{-}16$$

The second term on the right hand side of equation 3-16 denotes the RV estimator that is based on high frequency price observations. This estimate is corrected by a correction factor $\bar{n}$, where $\bar{n} = \frac{n-S+1}{S}$.

Figure 3-4 shows the TTSE estimates that are based on a single day for each of the East European indices during the observed period. From most indices the highest volatility was observed in the period 2011-2012. One exception is the .SOFIX index where the highest price jumps were realized in the period 2014-2015. In most cases the TTSE shows similarities over time across the different stock market indices. In most cases the second half of the observed period, 2013-2016, shows to be rather less volatile compared to the first half denoting 2010-2013.





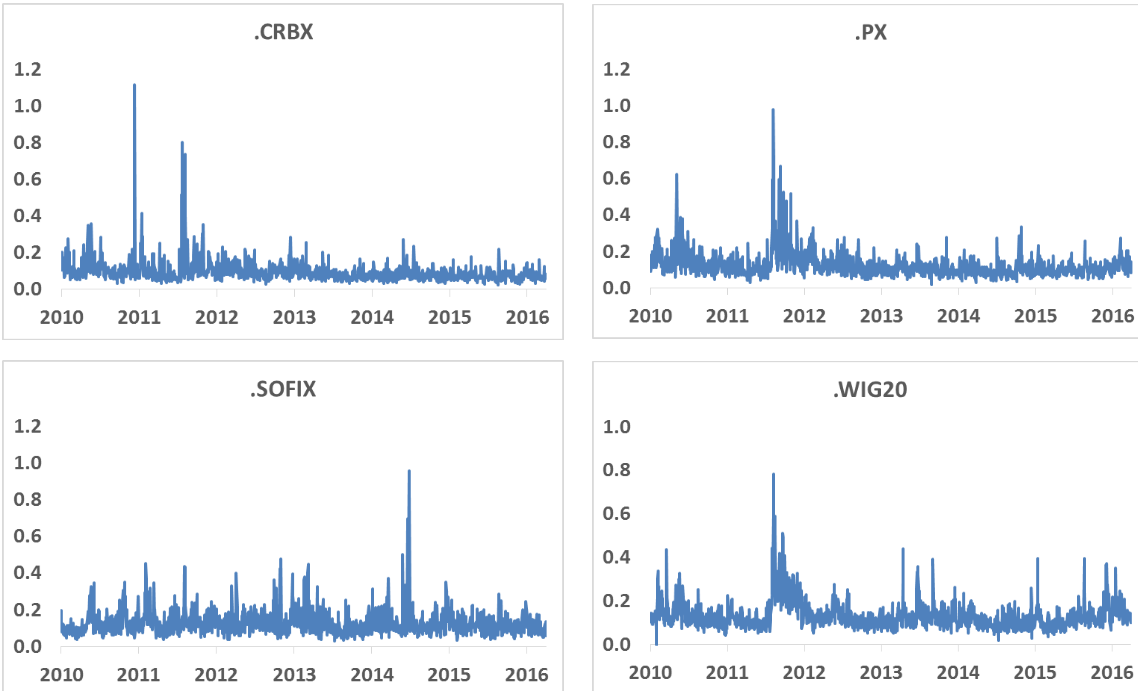


Figure 3-4 shows the TTSE estimates for the East European indices during the observed period.

The distribution of the returns of the indices are shows in figure 3-6 with corresponding statistics in table 3-6. Each of the indices show a skewness to the right of the distribution with some heavy tails on the far right. The sample sizes of the indices range between 1546 (.SOFIX) and 1576 (.BETI). The means of the distribution range between 0.093 (.CRBX) and 0.145 (.BUX).

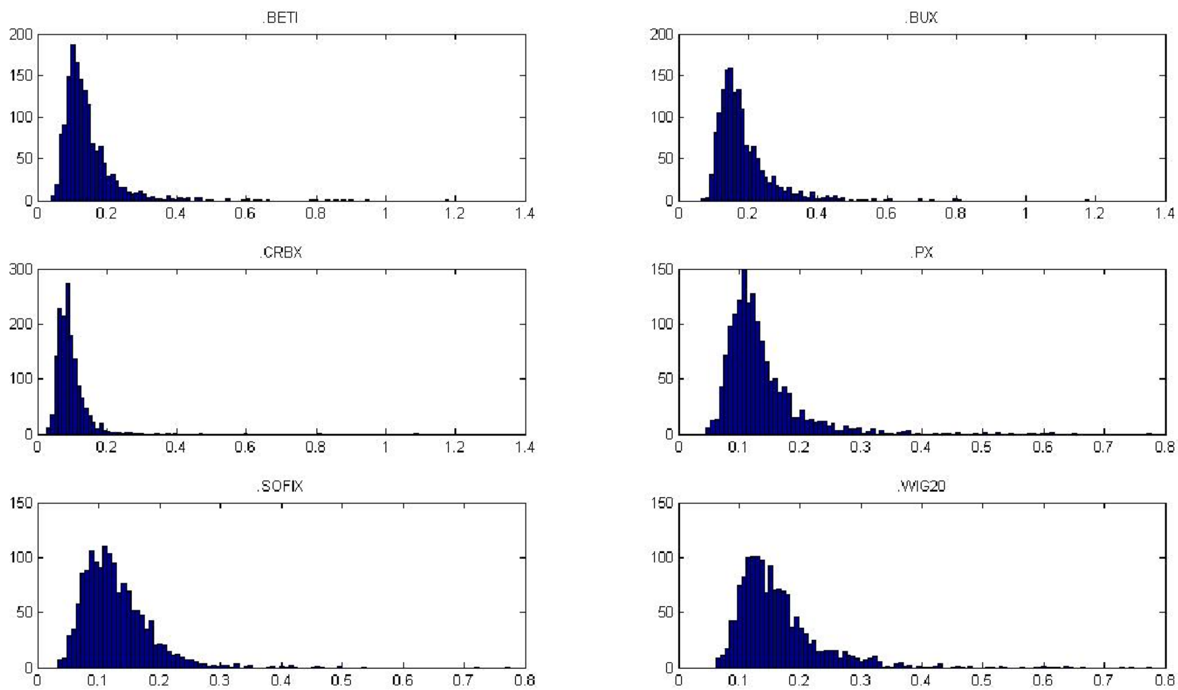


Figure 3-5 shows the historical distributions of the TTSE estimates for six East European indices.

The difference between the RV (Figure 3-1) and TTSE (Figure 3-4) can be prescribed to the microstructure noise that is present in the RV estimates. Figure 3-6 shows the difference between the TTSE and the RV estimates for each index. This difference has caused the "noise" in the RV estimator (equation 3-3) and is filtered out using the TTSE estimator (equation 3-16).

Table 3-3 showing statistical properties of TTSE estimates for the East European indices.

Index	.BETI	.BUX	.CRBX	.PX	.SOFIX	.WIG20
Mean	0.136	0.145	0.093	0.123	0.127	0.134
Median	0.111	0.129	0.082	0.107	0.114	0.119
Standard deviation	0.099	0.072	0.059	0.070	0.069	0.066
Skewness	4.603	2.926	6.969	3.960	3.472	2.864
Kurtosis	36.546	16.647	89.220	28.144	25.864	14.650
Sample Size	1576	1554	1562	1568	1546	1562

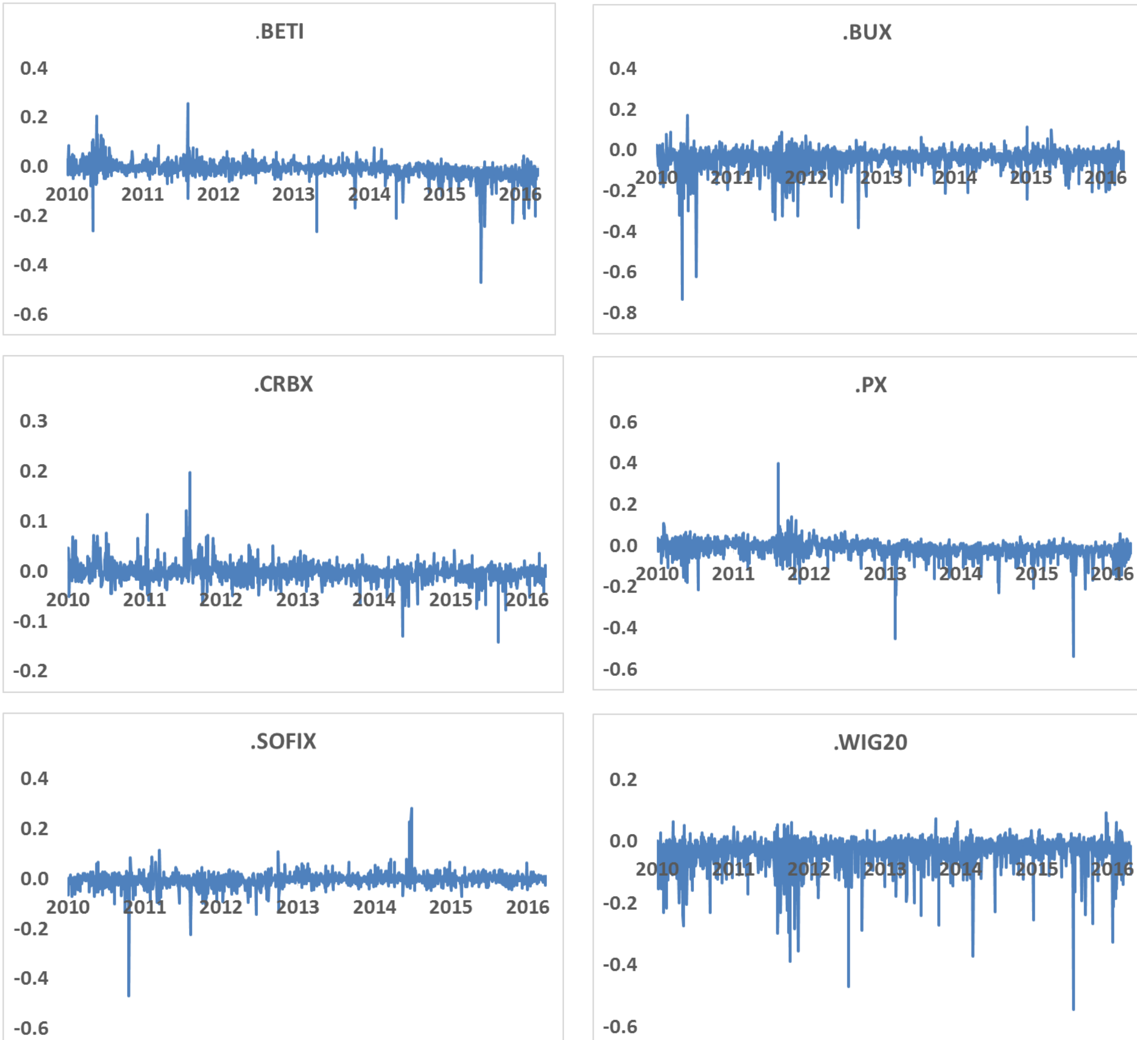


Figure 3-6 shows the differences between the RV and TTSE estimates for the East European stock indices.

Figure 3-7 shows the distributions of the differences between the RV and TTSE estimators for each of the stock indices. The distribution of the differences is concentrated around zero and shows skewness and tails on both sides of the distribution.

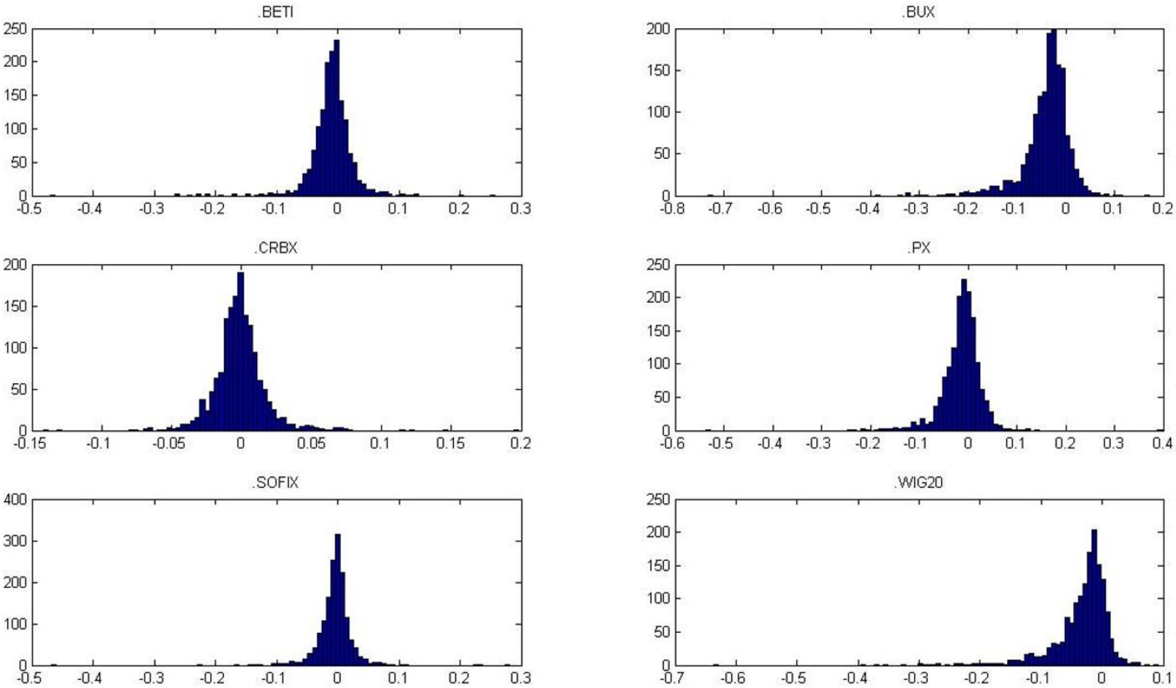


Figure 3-7 shows the distribution of the differences between the RV and TTSE estimates.

4 CHALLENGES IN ESTIMATING REALIZED VOLATILITY

Estimating Realized Volatility requires a model, modelling experience and data crunching skills. One of the biggest challenges in estimating Realized Volatility includes the enormous amount of intraday data that needs to be processed. These large data set may cause various challenges to the estimation process. Section 4.1 discusses the data quality challenges, which is occurs frequently, section 4.2 discusses the stylized facts about Realized Volatility and section 4.3 discusses the optimal choice of the sampling frequency.

4.1 Data quality and other limitation

The convergence relation in section 3.1 states that RV approximates IV arbitrarily well as the sampling frequency M increases. Three issues, however, complicate the application of this result.

First, continuous prices are unavailable. This theoretical assumption used in the theory of Realized Volatility requires some relaxation in practice. After all, there are only a few asset classes which reasonably comply with this extreme condition. Examples thereof are the foreign currency market, which trades around the globe continuously during workdays. Thus when the US market closes the trading continues on other foreign exchange markets as for example Tokyo, Sydney, etc. The only non-trading hours include weekends, i.e. from 22:00 GMT on Sunday (Sydney) until 22:00 GMT Friday (New York). Obviously this will only hold for the most liquid currencies (USD, EUR, JPY, AUD, GBP, etc.). For other asset classes, on the other hand, the trading hours are even less frequent as assets traded on the NYSE will, classically, not be traded at other Exchanges. This means that the trading hours depend on the particular Stock Exchange and will typically have 8 trading hours per day, which exclude weekends and national holidays. This is a limitation, which introduces a discretization error in the realized volatility measure.

Second, various microstructure effects contaminate the realized volatility estimates. Microstructure effects include negative autocorrelation, price discreteness and rounding, bid-ask bounces, etc.

Third, includes challenges caused by the limited availability of intraday data for particular assets or asset classes of interest. For example, the intraday database should consist of sufficient intraday data records at the required high frequency level. Data quality issues can limit the applicability of the theory of Realized Volatility for specific assets or asset classes. Infrequent intraday trading activities can also contaminate the high and low prices as was noted by Garman and Klass (1980). When intraday trading activities are infrequent the observed daily extremes could be less than the true extreme prices, which influences the volatility estimates.

4.2 Stylized facts about Realized Volatility

Realized Volatility is a discrete time intraday volatility model that estimates the actual volatility with high precision when the frequency of returns increases to infinite and the conditions outlined in section 3.1 apply. In practice intraday price observations are not continues and hence suffer from discretization error. A wide range of microstructure noise are commonly introduced while dealing with high frequency returns. As a result measurement error has become a consistent empirical finding of realized volatility estimates.

Intraday price observations can suffer from a wide range of microstructure noise induced by, for example, the bid-ask bounce, diurnal effects, persistency in volatility, volatility jumps, discreteness of price changes, etc. Microstructure noise is often considered a general definition that includes any type of shock that contaminates the IV. There are a number of stylized facts of RV that are consistent across empirical findings. This section describes a number of stylized facts of the RV estimator that are well known in the literature.

4.2.1 Negative Serial Correlation

Serial correlation is an important empirical finding that may impact the efficiency of the realized volatility estimator. It appears when error terms from different time periods are correlated as can be expected when high frequency intraday data is collected repeatedly across time. LeBaron (1992) studied the negative relation between serial correlation and volatility, also known as the "LeBaron effect", which manifests at daily and weekly levels. He found that serial correlation changes over time and that it is related to the stock return volatility. Bianco, Corsi and Reno (2009) showed that there is also a negative relation between serial correlation and volatility on intraday level, whereas Oomen (2012) shows that the realized volatility measure becomes biased when returns are serially correlated. A careful choice of the optimal sampling frequency can reduce the impact of serial correlation on the realized volatility measure to a very small amount. In line with Oomen (2012) we show how serial correlation manifests under certain conditions. Using the additive property of logarithmic returns, it follows that the daily return is an aggregation of *M* intraday returns, which can be written as

$$r_{t,\Delta} = \sum_{j=1}^{M} r_{t,j,\Delta} \tag{4-1}$$

The variance of $r_{t,\Delta}$ equals the sum of variances if and only if the intraday returns are uncorrelated. On the other side, when intraday returns are correlated, the variance of the sum of intraday returns equals the sum of the variances plus a covariance of intraday returns, i.e.

$$Var\left(r_{t,\Delta} \middle| F_{t-1}\right) = Var\left(\sum_{j=1}^{M} r_{t,j,\Delta} \middle| F_{t-1} \right) = \sum_{j=1}^{M} Var\left(r_{t,j,\Delta} \middle| F_{t-1}\right) + 2\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} Cov\left(r_{t,i,\Delta} r_{t,j,\Delta}\right) \tag{4-2}$$

, where *Var(x)* denotes the variance of variable *x*, *Cov(x,y)* the covariance between *x* and *y*, and F_{t-1} denotes the information set from the entire sample path of r_t available up to time *t-1*. The second term on the right hand side denotes the covariance of the intraday returns. Decomposing the squared return by means of the additive property leads to the sum of squared returns and a cross product denoted as the serial correlation

$$r_{t,\Delta}^2 = \left[\sum_{j=1}^{M} r_{t,j,\Delta} \right]^2 = \sum_{j=1}^{M} r^2_{t,j,\Delta} + 2\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} r_{t,i,\Delta} r_{t,j,\Delta} \tag{4-3}$$

Hence the volatility of r_t becomes equal to $\sum_{j=1}^{M} r^2_{t,j}$ if and only if the cross product on the right hand side is zero in expectation, i.e. $E\left(\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} r_{t,i,\Delta} r_{t,j,\Delta}\right)=0$. However when the serial correlation is not equal to zero the realized volatility becomes biased and over- or underestimates the actual volatility. Oomen (2004) shows in an empirical analysis on the FTSE-100 index that the impact of serial correlation on the realized volatility increases with higher intraday frequencies. At the 1 minute sampling frequency the bias due to serial correlation was estimated at 35%, while the optimal sampling frequency was between 25 and 35 minutes.

4.2.2 The bid-ask bounce

The bid-ask bounce model was initially introduced by Roll (1984) who used the model to justify the phenomenon that lower frequencies are considered to be less contaminated then the higher ones. In this model the bid and ask prices are set by dealers, while the bid-ask bounce describes a symptom when transaction prices "bounce" back and forth between the bid and ask prices. Even when intraday price observations continuously bounce between the bid and ask prices and never exceed its boundaries it might significantly impact the realized volatility estimate. We further elaborate on equation (4-3) assuming the same diffusion process as was assumed in equation (3-1). As was the case with negative serial correlation, microstructure noise is also not considered to be time invariant due to the bid-ask bounce, but will rather evolve over time.

Assume the bid-ask bounce is *i.i.d.*, with $E[\nu_t]=0$, $E\left[\nu_t^2\right]=\sigma_\nu^2$ and $E\left[\nu_t^8\right]<\infty$ and that the squared return series are also contaminated with microstructure noise denoted by $\nu_{t,j,\Delta}$. The sum of squared intraday returns $r^2_{t,\Delta}$ then takes the form

$$r^2_{t,\Delta}=\left[\sum_{j=1}^{M} r_{t,j,\Delta}\right]^2=\sum_{j=1}^{M} r^2_{t,j,\Delta}+2\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} r_{t,i,\Delta} r_{t,j,\Delta}\nu_{t,i,\Delta}\nu_{t,j,\Delta}+\sum_{j=1}^{M}\nu_{t,j,\Delta} \qquad 4\text{-}4$$

Thus, including serial correlation and microstructure noise due to the bid-ask bounce, the measurement error in RV is assumed to increase.

4.2.3 Intraday diurnal pattern

The intraday diurnal pattern aims to explain the relationship between intraday trade volumes and intraday price changes over time. The intraday trading volume and price changes often show a diurnal pattern during opening trading hours. In a “U-shaped” diurnal pattern the volume of trades and price changes are often higher during the beginning and the end of the trading day, while during lunch time it shows lowest trading volumes and price changes. A “decreasing” or “increasing pattern” is similar to a “U shaped” except for having a lower trading volume at the beginning, respectively at the end of the trading day. Although not common in practice, an “inverted U shaped” diurnal pattern characterizes low trading volumes during the beginning and the end of the trading day, while during lunch time the trading volume increases. Figure 4-1 shows a graphical example of the 4 diurnal patterns described.

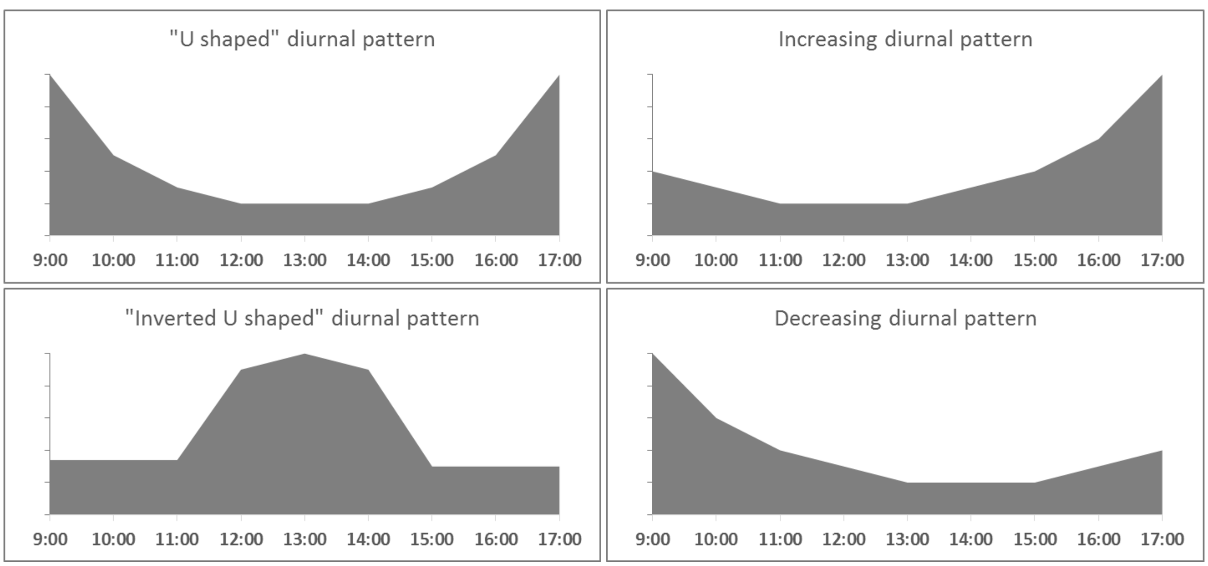


Figure 4-1 Theoretical examples of diurnal effects showing the volume of trades on the vertical side and the intraday trading hours on the horizontal axes.

According to Liu and Maheu (2011) intraday diurnal patterns also have a repeating character, but do not tend to manifest in a daily, weekly or monthly seasonal affect. The volatility estimates in scope of this thesis are, at least, on a daily basis. For this reason daily volatility estimates are not considered to be sensitive to intraday diurnal patterns.

The trading volume during trading hours of each index is shown in Figure 4-2. Most of the indices show a “U” shaped pattern in the middle of the day, while the trading activity decreases during trading hour. Only exception is the index, which shows “peak” around 12:00h indicating a slight increase in trading activities during lunch time. Another exception is the trading activity during the beginning of the trading day. Notice that both CRBX and BETI start with lower recorded trading activities, while WIG, BUX and PX show much higher overnight trading activities.

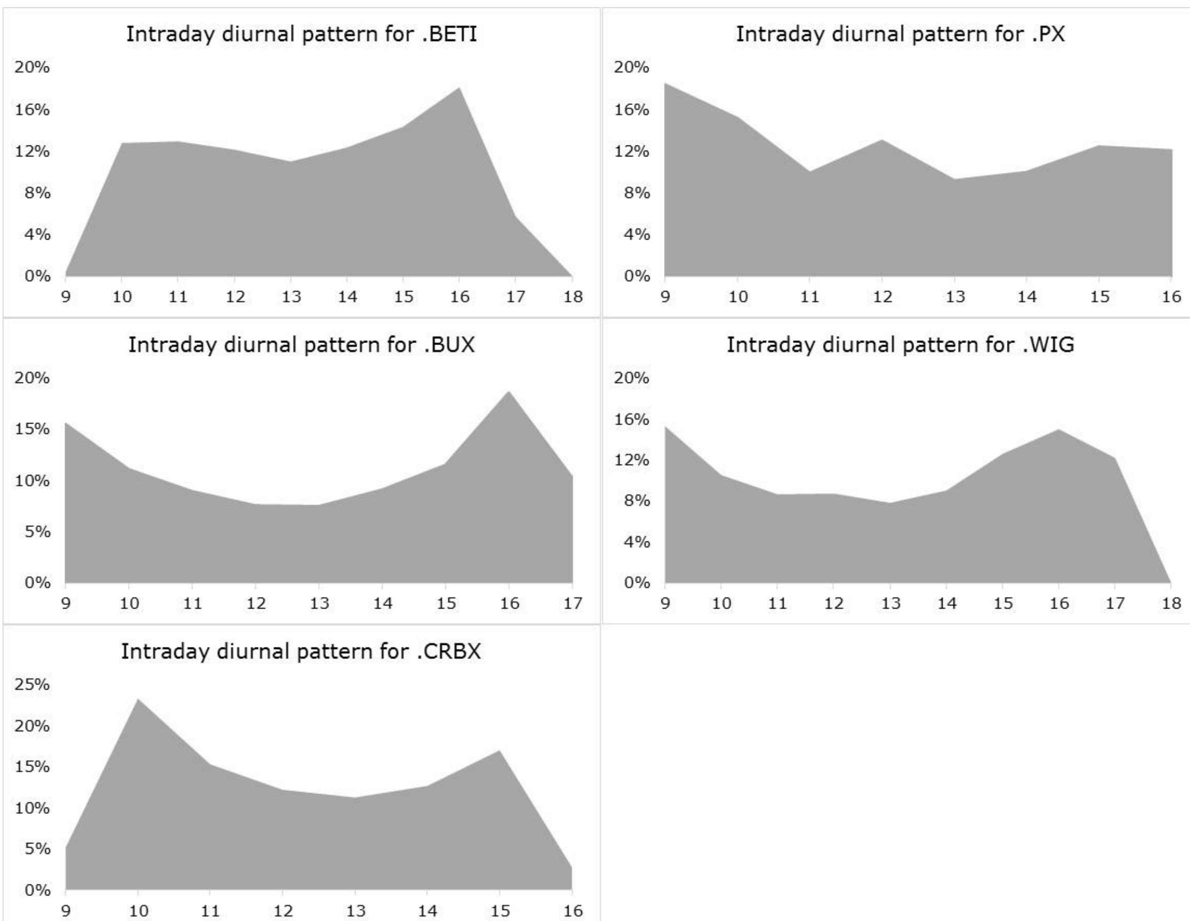


Figure 4-2 Intraday diurnal patterns based on empirical data of the selected indices.

4.2.4 Volatility jumps

Assets prices may exhibit sudden discrete price movements when unexpected news reaches the market. Assume the semi-martingale process in equation (3-1) includes jumps (J) and is defined by

$$r_t^h = \int_t^{t+h} \mu(u)du + \int_t^{t+h} \sigma(u)dWu + \int_t^{t+h} J(u)du \qquad 4\text{-}5$$

The IV over the interval t to $t+h$ becomes

$$IV_t^{(h)} = \int_t^{t+h} \sigma^2(u)du + \sum_{t \le u \le t+h} J^2(u) \qquad 4\text{-}6$$

where J(t) is non-zero only if there is a jump at time *t.*

The IV of the semi-martingale process in equation (3-1) can also be estimated with the Realized Bipower Variation (BV), which provides a consistent estimate of the IV. In its simplest form BV can be defined by

$$BV_{t,\Delta} \equiv \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=1}^{M-1}\left|r_{t,j,\Delta}\right|\left|r_{t,j+1,\Delta}\right|, \qquad 4\text{-}7$$

where $r_{t,j,\Delta}$ is the j intraday stock price on day t, $j \in (t = 1,..., M)$ and Δ is the sampling frequency. Barndorff-Nielsen and Shephard (2004) show that the BV estimate converges to the same probability limit as RV when there are no jumps in the semi-martingale process. Hence the jump in the process is defined by

$$J_{t+1,\Delta} \equiv \max\{RV_{t,\Delta} - BV_{t,\Delta}, 0\}. \qquad 4\text{-}8$$

4.2.5 Leverage effect and volatility feedback effects

Leverage and volatility feedback effects are a phenomenon that describe an often observed asymmetry in volatility estimates. Namely, negative equity returns tend to increase the volatility more often than positive ones. This phenomenon was first noticed by Black (1976) and is commonly referred to as Black's leverage hypothesis, i.e. "*the tendency of negative correlation between the return and volatility of equities*".

A decrease in the market value of an equity increases the debt-to-equity ratio and results in a relative increase of the leverage of the underlying company. Higher leverage also increases the risk of bankruptcy and often translates to a higher risk perception resulting in an increase of the volatility of the underlying stock price. A leverage effect with reversed causality is denoted as the volatility feedback effect, i.e. the tendency of negative correlation between volatility and return of equities.

4.3 Sampling frequency selection

Microstructure noise contaminates the results of the realized volatility estimate. A simple, yet efficient solution to reduce the impact of microstructure noise is to use sampling at arbitrary selected lower frequencies as stated in Andersen et al. (2001). Equidistant sparse sampling frequencies, of which the most popular frequencies lay between 1 and 60 minutes, instead of sampling at extremely high frequencies can reduce the existence of microstructure noise.

Assuming a weakly stationary return process as denoted in equation (3-1) the average of both terms on the right hand side of the RV (equation 3-1) denote the average realized volatility ($\overline{RV}$) and the average serial correlation factor ($\overline{\Omega}$) respectively.

$$\overline{RV_{\Delta}} = \frac{1}{T}\sum_{t=1}^{T}\left(\sum_{j=1}^{M} r^2_{t,j,\Delta}\right) \tag{4-9}$$

$$\overline{\Omega_{\Delta}} = \frac{1}{T}\sum_{t=1}^{T}\left(\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} r_{t,i,\Delta} r_{t,j,\Delta}\right) \tag{4-10}$$

The optimal sampling frequency selection is based on a bias-variance trade-off. The choice is between the precision, which is based on the sampling frequency, and the incurring bias. A higher frequency would result in more precision, but with a larger bias. Whereas a lower frequency would result in less precision, but with a lower bias. The so called “volatility signature plot”, as suggested by Andersen et al. (1999), plots the average RV (eq. 4-10) against the average serial correlation (eq. 4-11) for intraday sampling frequencies ranging from 1 minute to 120 minute.

Andersen and Bollerslev (1998) and Andersen et al. (2001) where amongst the first to propose the 5-minute sampling frequency for measuring the volatility on the foreign exchange market. Ait-Sahalia et al. (2005), Zhang et al. (2005), Hansen and Lunde (2006) and Bandi et al. (2008) propose different optimal sampling frequencies for different assets. It is more common to propose a higher frequency, of for example 1 minute, for liquid assets, while for illiquid assets a much lower frequency would be common.

4.4 Summary of the challenges in Estimating Realized Volatility

Estimating (realized) volatility by means of (ultra HF) intraday data is not without any challenges. The first challenge includes the data quality of the asset and asset class of interest. Data filtering techniques might be required to "clean" the data. This also may include the optimal sampling choice and the choice of the model to estimate the (realized) volatility. Finally analytical skills are required to analyse the results.

This section discussed the stylized facts of realized volatility. At the (ultra) high frequency the distribution of RV is empirically strongly skewed to the right and has a very high Kurtosis reflecting high outliers. On the other side, at the lower sampling frequency (e.g. OHLC data), the distribution of RV is less skewed to the right and has a lower kurtosis denoting the robustness of the optimal sampling frequency versus outliers. Positive and negative innovations impact the realized volatility asymmetrically. This is also known as leverage effect. Next to this volatilities are time varying and clustering:

1. Although the return distribution is non-Gaussian and leptokurtic, the standardized return (the ratio of return to realized standard deviation) distribution conforms well to the normal distribution.
2. While neither realized daily variance nor realized daily standard deviation follow the normal distribution, the distribution of realized daily log-variance is closer to the normal distribution.
3. Standard unit root tests often reject the presence of a unit root in realized daily variance.

As volatility is commonly known to be mean reverting meaning that current information has zero impact on the long run forecast.

5 EXTENDING RANGE-BASED VOLATILITY ESTIMATES

Range-based volatility estimators that only utilize OHLC data belong to the set of low-frequency volatility estimators. The estimates require only a limited amount of intraday observations consisting of a combination of the open, high, low and closing prices. Low frequency data has the practical advantage of being widely available across a vast amount of asset classes and markets. Besides this, volatility estimators that are based on low-frequency data also have in common that they are computationally less complex then estimators that require high-frequency data. This chapter discusses existing models for estimating volatility based on only low-frequency data, it investigates the impact of overnight returns on these models and proposes an extension of the standard low frequency models by including the overnight price changes.

5.1 Overnight returns

Asset prices of financial exchange markets are typically only known at discrete time frequencies during official trading hours. The length of a trading day of an exchange market will depend on various factors. The type of the market will influence the length of the trading day as stock exchange markets typically close earlier than foreign currency exchange markets. The geography of the exchange market also plays an important role because of national holidays and other national influences that determine the trading hours. Stock exchange markets have typically around eight trading hours during a trading day. Some of the Asian stock exchange markets also have a lunch break around 12:00 pm local time. A typical example of a 24-hour market is the foreign exchange market which trades around the globe and begins on Sunday at 5:00 pm ET in Sidney and ends at Friday 5:00 pm ET in New York. Futures markets also have typically longer trading days than stock exchange markets. It ranges between

8 and 23 hours per trading day depending on the particular futures market.

Opening prices are the first publically available trading prices after a non-trading period and Closing prices are the last publically available trading prices of a trading period. Stock exchange markets will typically have five opening and closing prices during a week, assuming there are no trading breaks during the day. On the other side FX markets of leading currencies, e.g. EUR/USD and GBP/USD, will typically have only one open and close price during a week as the trading continues around the globe.

Opening prices will be rarely equal to closing price of the previous trading day as new publically available information can influence the supply and demand for any particular security. The market reacts by ordering a buy or a sell of the underlying security. If the information was released outside of trading hours then the buy or sell order will be executed at the opening of the exchange market and thus influence the opening price. If the demand increases in comparison to supply than the price will increase and, vice versa, if the demand decreases in comparison to the supply than the price will decrease. Next to publicly available information, future markets can continue trading during the closing period of the underlying exchange markets. The value of futures will generally impact the price of the underlying assets, which will be reflected in the opening price. Hansen and Lunde (2006) argued that a proper proxy for volatility should not only include the information available during trading hours, but should instead include all available daily information to cover a full period of 24-hours. To achieve this goal the overnight returns need to be included in the estimator.

The difference between the opening price and the closing price of the previous day is defined as the opening jump. Opening jumps can largely impact the volatility estimation. Opening jumps, γ_t, for the entire sample period of the six European indices (.BETI, .BUX, .CROBEX, .PX, .SOFIX and .WIG). The logarithmic difference between the opening price and the previous closing price is formulated by $\gamma_t = \left(\ln \frac{O_t}{C_{t-1}} \right)$.

All opening jumps are equally weighted as was earlier proposed by Bollerslev et al. (2009), de

Pooter et al. (2008), Becker et al. (2007), Martens (2002) and Blair et al. (2001).

Table 5-1shows the descriptive statistics of the opening jumps for all indices covering the entire sample path. For the .BETI and .SOFIX the opening prices equal the closing prices of the previous trading day and therefore there are no opening jumps observed in the descriptive statistics. Opening jumps are observed for .CRBX, .BUX, .PX and .WIG indices, which show a standard deviation ranging between 0.20% and 0.60%. The average ranging between 0.00% and 0.04% indicate historically positive expectations in opening jumps. The negative skewness for these indices indicate that there is a higher probability of extreme negative opening jumps then extreme positive opening jumps.

Table 5-1 Descriptive statistics of opening jumps covering the period January 2010 to April 2016.

4/1/2010 - 1/4/2016	.BETI	.BUX	.CRBEX	.PX	.SOFIX	.WIG20
Count	1576	1554	1557	1567	1550	1563
Min	-	-4.74%	-5.50%	-4.04%	-	-4.85%
Max	-	6.20%	0.74%	3.71%	-	3.30%
Mean	-	0.04%	0.00%	0.02%	-	0.03%
St.Dev	-	0.59%	0.20%	0.46%	-	0.60%
Skewness	-	-0.22	-22.74	-0.45	-	-0.82
Kurtosis	-	16.43	596.36	17.35	-	7.44

Tables 5-2 and 5-3 show the descriptive statistics for two sub periods, i.e. until February 15th of 2013 and from February 15th 2013 onwards. The statistics indicate that the average, standard deviation and skewness in the opening jumps were lower in the second period. The only exception being .WIG20, which shows an increase in the average, the skewness and kurtosis.

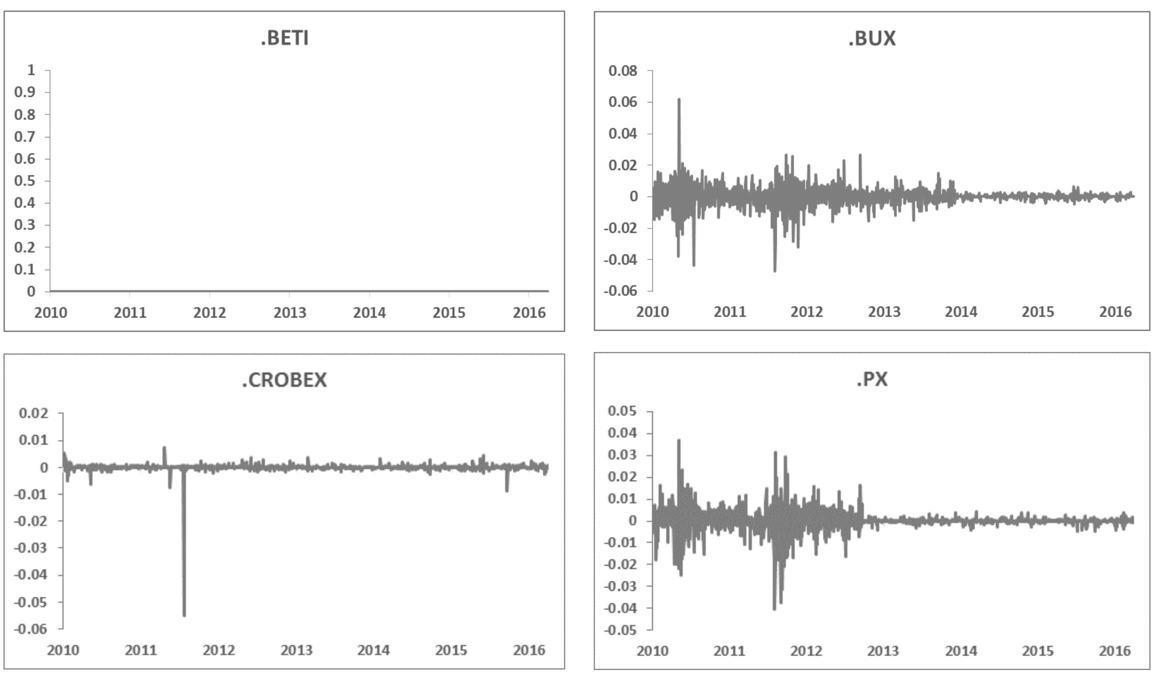
Table 5-2 Descriptive statistics of opening jumps covering the period January 2010 to February 2013.

4/1/2010 - 15/2/2013	.BETI	.BUX	.CRBEX	.PX	.SOFIX	.WIG20
Count	786	786	786	786	786	786
Min	-	-4.74%	-5.50%	-4.04%	-	-4.85%
Max	-	6.20%	0.74%	3.71%	-	3.30%
Mean	-	0.06%	-0.01%	0.05%	-	0.02%
St.Dev	-	0.79%	0.27%	0.64%	-	0.71%
Skewness	-	-0.24	-17.47	-0.45	-	-0.65
Kurtosis	-	8.61	334.00	7.73	-	5.63

Table 5-3 Descriptive statistics of opening jumps covering the period February 2013 to April 2016.

15/2/2013 - 1/4/2016	.BETI	.BUX	.CRBEX	.PX	.SOFIX	.WIG20
Count	790	768	771	781	764	777
Min	-	-1.51%	-0.87%	-0.48%	-	-3.66%
Max	-	1.50%	0.45%	0.41%	-	2.27%
Mean	-	0.03%	0.00%	0.00%	-	0.03%
St.Dev	-	0.24%	0.06%	0.09%	-	0.47%
Skewness	-	0.30	-2.18	-0.64	-	-1.17
Kurtosis	-	9.00	48.81	8.51	-	8.03

Figure 5-1 confirms the statistical observations for all indices. As expected the .BETI and .SOFIX indices don’t show any overnight jumps in the entire history. For .BUX and .PX it is obvious from the figures that during the second half, the observations show lower opening jumps in absolute terms. The .CRBX shows volatility of the overnight returns during the entire period. The .CRBX also shows some downward outliers between 2011 and 2012 and between 2014 and 2015.



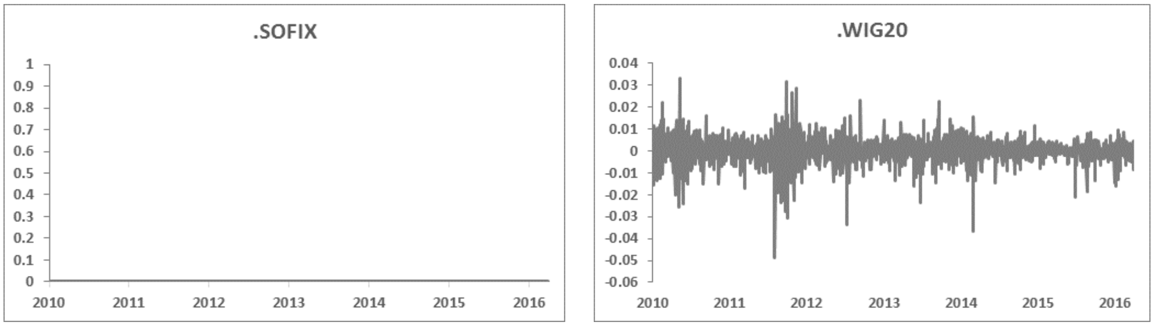


Figure 5-1 shows the overnight jumps of the East European stock indices.

5.2 Extended range-based volatility estimates

Based on the idea of Yang and Zhang (2000) opening jumps can be added to classical OHLC volatility estimators to allow for, upward and downward, opening jumps. The expectation is that adding an extension which allows for opening jumps will have a significant impact on the performance of the classical volatility estimators of stock assets if these opening jumps are significant.

A straightforward solution is to linearly extend the estimators by the logarithmic difference between the opening price on day t and the closing price on the previous day. Assume that the extension that allows for opening jumps with respect to the previous trading day takes the form: $\left(\ln\frac{O_t}{C_{t-1}}\right)^2$. The following extended estimators are defined:

1. The extended Close-to-Open takes the form:

$$\hat{\sigma}^2_{t,C} = \left(\ln\frac{C_t}{O_t}\right)^2 + \left(\ln\frac{O_t}{C_{t-1}}\right)^2 \qquad 5\text{-}1$$

The extended Parkinson, Garman & Klass and Roger & Satchell take the following forms:

2. Extended Parkinson

$$\hat{\sigma}^2_{t,P*} = \frac{1}{4\ln(2)}\cdot\left(\ln\frac{H_t}{L_t}\right)^2 + \left(\ln\frac{O_t}{C_{t-1}}\right)^2 \qquad 5\text{-}2$$

3. Extended Garman & Klass

$$\hat{\sigma}^2_{t,GK*} = \frac{1}{2}\left(\ln\frac{H_t}{L_t}\right)^2 - (2\ln 2 - 1)\cdot\left(\ln\frac{C_t}{O_t}\right)^2 + \left(\ln\frac{O_t}{C_{t-1}}\right)^2 \qquad 5\text{-}3$$

4. Extended Roger & Satchell

$$\hat{\sigma}^2_{t,RS*} = \ln\frac{H_t}{C_t}\ln\frac{H_t}{O_t} + \ln\frac{L_t}{C_t}\ln\frac{L_t}{O_t} + \left(\ln\frac{O_t}{C_{t-1}}\right)^2 \quad 5\text{-}4$$

5. Extended High-Low

$$\hat{\sigma}^2_{t,HL^*} = ln\left(\frac{H_t}{L_t}\right)^2 + \left(ln\,\frac{O_t}{C_{t-1}}\right)^2 \quad 5\text{-}5$$

Table 5-4 gives an overview of the extended models:

Table 5-4 Overview of the extended range-based volatility models

Volatility estimate		**Prices taken**	**Include previous day prices**	**Include Drift**	**Include o/n jumps**	**Theoretical efficiency gain**
$\hat{\sigma}^2_{i,coc}$	Close-to-Open-to-Close	OC	Yes	No	Yes	Yes
$\hat{\sigma}^2_{i,Pext}$	Parkinson Extended	OHLC	Yes	No	Yes	Yes
$\hat{\sigma}^2_{i,GKext}$	Garman-Klass Extended	OHLC	Yes	No	Yes	Yes
$\hat{\sigma}^2_{i,RSext}$	Roger-Satchell Extended	OHLC	Yes	No	Yes	Yes
$\hat{\sigma}^2_{i,HL}$	High-Low Extended	KL	Yes	No	Yes	Yes

6 Theory Of Ranking Volatility Estimates

There is a wide range of range-based volatility estimators available in the literature, with each of them having their own characteristics and specificities. Range-based volatility estimators can differ significantly from each other according to Duque and Paxson (1997) who also concluded that the choice of the range-based volatility estimator is important. Whilst the choice of the estimator is important for the analysis, the correct ranking methodology that will be used to find the most appropriate volatility estimator is the first step to take before ranking a set of competing estimators. It's worthwhile noticing that different ranking methodologies do not necessarily guarantee the same unique result. The choice of the ranking methodology is, however, important and should take into account the ultimate purpose of the estimator. For example, if the ultimate purpose would be to measure the extremes, it might be useful to have a suitable ranking methodology that gives the highest ranking score to the estimator that is most suitable for the purpose.

The first three sections cover the most popular ranking methodologies found in the literature and provide useful new insight in existing ranking methods. Section 6.1 discusses the classical efficiency coefficient, which has continually been used in a large number of previous research and which can be seen as a general method for cross-literature comparison of the ranking results. This section adds value by replacing the close-to-close estimator, which is the benchmark in the classical efficiency coefficient, with the unbiased benchmark, i.e. TTSE. Section 6.2 describes the Mincer-Zarnowitz linear regression model, which investigates the linear relationship between the volatility benchmark and the competing volatility estimates. More recent literature on ranking of volatility estimates advocates the use of the loss function approach, which is discussed in section 6.3. Among a wide range of available loss functions only a small set of robust loss functions are exploited and used for ranking.

Section 6.4 exploits a Copula function approach for ranking volatility estimates. This is a more suitable ranking methodology for when the extremes of the volatility estimates are of importance. Although different ranking methodologies do not guarantee the same unique results, they can be used in isolation for a specific purpose or in a joint assessment for a general overview.

6.1 Ranking based on the efficiency coefficient

The classical efficiency coefficient is a metric that has been used to rank range-based volatility estimates in a vast number of literature on estimating volatility with range-based models. By continually using the same methodology it has been possible to compare the results across a wide range of literature. Examples hereof include the main contributors of this specific literature: Garman & Klass (1980), Parkinson (1980) and Rogers & Satchell (1991). The standard benchmark proxy used in the early literature was the close-to-close volatility estimator, which on the one hand made the results comparable across the literature, but on the other hand is a biased estimator of the true, integrated volatility that can contaminate the ranking analysis. In the classical efficiency coefficient, the variance of the competing, range-based volatility estimator, $\hat{\sigma}_i^2$, is compared against the variance of the close-to-close volatility estimator, $\hat{\sigma}_c^2$:

$$Eff_i^{\,c}\left(\hat{\sigma}_i^2\right)=\frac{\mathrm{var}\left(\hat{\sigma}_c^2\right)}{\mathrm{var}\left(\hat{\sigma}_i^2\right)}, \qquad 6\text{-}1$$

where Var(x) denotes the variance of x. The close-to-close estimator is merely an approximation of the true, unknown and unbiased volatility estimator. With the availability of intraday data and recent insight in the theory of realized volatility, the close-to-close estimator can be replaced as a benchmark with the unbiased, $\hat{\sigma}_{TTSE}^2$, volatility estimator. The new efficiency estimator then takes the form:

$$Eff_i^{\,TTSE}\left(\hat{\sigma}_i^2\right)=\frac{\mathrm{Var}\left(\hat{\sigma}_{TTSE}^2\right)}{\mathrm{Var}\left(\hat{\sigma}_i^2\right)} \qquad 6\text{-}2$$

In both cases the efficiency coefficient will equal one when the volatility estimator, $\hat{\sigma}_i^2$, equals the benchmark estimator, $\hat{\sigma}_B^2$. Where the B stands for the benchmark estimator, which is either the close-to-close (equation 6-1) or the TTSE volatility estimator (equation 6-2). The efficiency coefficient will either become close to zero or explode in value if the competing volatility estimator differs extremely from the benchmarking model. In other words, the closer the efficiency coefficient gets to zero, the more the competing estimator equals the benchmark. The efficiency coefficient can therefore be seen as a suitable metric for overall comparison and ranking of range-based volatility estimates.

A downside about these two metrics is that they provide input for the ranking methodology based on only a single value. Namely, the variance of both, the nominator and denominator, provide a very general view on the characteristics and specificities of the competing estimators. It does not provide information for comparison that is based on specific levels or movements of the benchmark volatility during time. For example, the competing estimator might move in the opposite direction of the benchmarking model and thereby give a complete wrong indication of the true volatility at that specific moment in time. The efficiency coefficient is also not a suitable metric for determining the correlation of extreme movements, as the efficiency coefficient might assign a high score to a competing estimator, while during extreme movements the competing estimator fails to capture the true volatility.

6.2 Mincer-Zarnowitz regression

Correlations have an explanatory property that explains the influence of variables. Correlations provide more explanatory power and include more information than for example a simple efficiency coefficient. Both the direction of the volatility estimator and the magnitude are of importance in the selection process. In line with this statement Kayahan, Saltoglu and Stengos (2002) use the coefficient of determination resulting from the Mincer-Zarnowitz regression to show that the 5-minute frequency Istanbul Stock Exchange provides a better fit than the normal GARCH model. The Mincer-Zarnowitz regression, see Mincer-Zarnowitz (1969), is a popular method for assessing the performance of volatility models. It describes the linear relationship between the competing, range-based, volatility estimates and the volatility benchmark. It is a linear regression of the unbiased volatility estimate, $\hat{\sigma}^2_{TTSE}$, against each of the n competing range-based volatility estimates, $\hat{\sigma}^2_n$. The Mincer-Zarnowitz regression is of the form

$$\hat{\sigma}^2_{TTSE} = \beta_0 + \beta_1 \hat{\sigma}^2_n + u_n \,, \qquad \text{6-3}$$

where β_0 denotes the intercept, β_1 the slope and u the error term. The coefficient of determination of the regression, $R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$, where SS_{res} is the Sum of Squares of the residuals of the linear equation (equation 6-3) and SS_{tot} denotes the Sum of Squares of the totals, which is proportional to the variance of the linear regression. The higher the coefficient of determination, the higher the likelihood between the competing volatility model and the benchmark. Thus $\hat{\sigma}^2_{TTSE} = \hat{\sigma}^2_n$ when $R^2 = 1$.

Previous literature on volatility estimation has often used the daily squared return as a benchmark in absence of the true unbiased volatility estimator. This has usually resulted in relatively low numbers of the coefficients of determination. Andersen and Bollerslev (1998) demonstrated that a low coefficient of determination is likely due to the fact that the daily squared return is a very noisy proxy for the (unobservable) variance. We solve this problem by replacing the daily squared return with the unbiased volatility estimator, TTSE.

The Mincer-Zarnowitz regression, however, is linear and therefore assumes that the correlation is constant over time. Since one of the stylized facts of financial time series states that dependence is not constant over time and that it moves countercyclical, see for example: Cont et.al. (2001), the results of the Mincer-Zarnowitz regression should be taken with additional care.

6.3 Loss function approach

Loss functions are one of the most popular ranking methodologies that can be found in the literature. They are often used to compare the predictive performance of competing models. Similar to previous research that focused on the predictive performance of volatility estimators, see Lunde (2005), Laurent, Rombouts and Violante (2009) and Patton (2011), this research considers loss functions to rank competing estimators against the benchmarking volatility model. The results of a statistical loss function are used to rank the models in accordance from best to worst. Patton (2011) finds that both the Mean Squared Error (MSE) and the Quasi

Likelihood (QLIKE) loss functions belong to the class of robust loss functions when used for comparing volatility models. This means that both MSE and QLIKE will provide consistent ranking even if an inefficient benchmark is used instead of the unbiased volatility estimator.

The competing range-based volatility estimates, $\sigma_{t,i}$, are the imperfect proxies of the true volatility, $\sigma_{t,TTSE}$. The overall distance between $\sigma_{t,TTSE}$ and $\sigma_{t,i}$, is measured by the mean square error (MSE), which is of the form:

$$MSE = n^{-1}\sum_{t=1}^{n}\left(\sigma_{t,TTSE} - \sigma_{t,i}\right)^2 \qquad \text{6-4}$$

The QLIKE loss function is of the form

$$QLIKE = n^{-1}\sum_{t=1}^{n}\left(\log(\sigma_{t,TTSE}^2) + \sigma_{t,i}^2 / \sigma_{t,TTSE}^2\right) \qquad \text{6-5}$$

Loss functions are suitable for a general view on the volatility models. However, the approach is not the most suitable methodology when ranking the volatility estimates based on the risk that can be found in the tails of the return distribution.

6.4 Pearson Correlation Coefficient

The Pearson's Correlation Coefficient, also referred to as a linear correlation coefficient, is defined as the covariance of two variables divided by the product of the variances.
It is defined in the interval [-1, 1]. The correlation coefficient is a single value denoting the direction and magnitude of the correlation between the two variables. For example, a correlation of -1 would indicate two variables moving in opposite direction with the same magnitude, while a correlation of 1 would indicate two variables moving in the same direction with the same magnitude and a correlation of zero would indicate two variables that neither move in the same direction nor with the same magnitude.

The correlation coefficient between the benchmark model and each range-based volatility estimate, *i*, is denoted with $\rho\left(\sigma_{TTSE}, \sigma_i\right)$ defined as:

$$\rho\left(\sigma_{TTSE},\sigma_i\right)=\frac{Cov\left(\sigma_{TTSE},\sigma_i\right)}{\rho\left(\sigma_{TTSE}\right)\rho\left(\sigma_{TTSE}\right)}, \qquad 6\text{-}6$$

where *Cov(x,y)* denotes the covariance between the variables *x* and *y* and $\rho(x)$, denotes the variance of the variable *x*.

The advantage of this approach is that it provides a simplistic methodology for ranking range-based volatility estimates. However the coefficient is not considered robust to outliers and as such it would not easily detect the correlation in the tails of the distribution.

We calculate the Pearson's correlation coefficient for each of the volatility estimates against the benchmark model for the total period of the historical data, i.e. from January 2010 to April 2016 and for two sub periods, denoted by before and after February 2013.

6.5 Copula function - Tail dependence approach

The tail dependence approach for ranking volatility estimates measures the comovement of the estimates in the extremes of the distribution. In the bivariate case, the tail dependence measures the probability of an alternative volatility estimate exceeding a certain threshold given that the benchmark volatility estimate has already exceeded this threshold. For the ranking function the tail dependence approach can be seen as a powerful tool for measuring the dependence of the alternative volatility estimates with the volatility benchmark during periods of high volatility. This is an important ranking function when the focus of interest lays in the extreme events. Thus, when the alternative estimate should as precise as possible estimate the volatility during period of high volatility. This is usually not measured with the loss function or the Mincer-Zarnowitz regression approach. The tail dependence approach utilizes a Copula function, which is the joint distribution of an alternative volatility estimate and the volatility benchmark.

Copula functions are a powerful tool in modelling non-linear dependence structures between financial variables. Although Copula functions are considered as relatively new in financial applications its use has gained in popularity in the financial literature over the past few decades.

Although we have not recorded it’s applicability in ranking of volatility estimates, its range of applicability in finance is wide. For example, Li (2000) shows how Copula functions can be applied to determine the default correlation in collateralized debt structured fixed income products. Embrechts, McNeil and Straumann (1999) introduce Copula functions as a powerful concept to aggregate risk, Manistre (2008) shows the applicability of copula functions in aggregating economic capital and translates the results of the Copula function back to a static correlation matrix. A nice literature overview of Copula functions in econometric modelling, as suggested by Embrechts (2007), can be found in Patton (2009) and Genest (2009). The main interest of this research, however, is not in providing a general bibliometric overview as this has been described in many articles before. This research focusses on utilizing Copula functions to describe the non-linear dependence relationship of the extremes between range-based volatility estimates and the unbiased volatility estimator, i.e. the TTSE benchmark model. The literature on Copula functions in finance has mainly focused on volatility forecasting and portfolio optimization. For as far as we are aware there has not been an attempt in the literature to use Copula functions for determining the dependence between volatility estimates. The goal of this investigation is to show how tail correlations, an important product of Copula functions, can be used to rank the range-based volatility estimates. For example, the copula function of a bivariate distribution describes the dependence structure, which can be used for pairwise comparison.

An important advantage of the Copula based approach compared to, e.g., the Mincer-Zarnowitz regression or a Pearson correlation coefficient is that the Copula based approach allows for non-linearity in the dependence structure. The Pearson correlation coefficient provides a single number to determine the linear dependence between two variables, while the Copula function provides information about the correlation over the entire distribution of the variable. The extreme variances that can be found in the tail of the distribution often show non-linear dependence, which cannot be detected with a linear correlation coefficient such as the Pearson’s coefficient or the Mincer-Zarnowitz regression. A non-linear copula function provides information about the structure of the dependence in the extremes through the tail correlation, which is a product of the copula function.

In mathematical terms, let the variable X represent the unbiased volatility estimator *TTSE* and let variable Y represent one of the competing range-based volatility estimators. Sklar (1959) shows that any joint distribution function of random variables X and Y can be decomposed into its cumulative distribution functions $F_X(x)$ and $F_Y(y)$, which are also known as marginal distribution functions, and a Copula function.

A Copula function that combines the marginal distribution function can be mathematically expressed as: $F_{XY}(x,y) = P(X \le x, Y \le y) = C(F_X(x), F_Y(y))$. The marginal distribution functions $F_X(x)$ and $F_Y(y)$ are uniform transformed variates of u and v respectively, with $u, v \in [0,1]$. A bivariate copula, joint distribution, function of the uniform variates transforms the variables X and Y into uniform random variables, $U \sim [0,1]$ and $V \sim [0,1]$, is of the form: $C(u,v) = P(U \le u, V \le v)$. The Copula function links the univariate distributions with the following relationship: $C(u,v) = 0,\ C(u,1) = u$ and $C(1,v) = v$.

In general, it can be shown that the coefficient of the upper tail dependence measures, λ_u, can be obtained with a function of the copula $C(u,v)$ given by

$$\lambda_u = P(U > q \,|\, V > q) = \lim_{q \to 1-} \frac{1 - 2q + C(q,q)}{1-q} \qquad 6\text{-}7$$

A popular Copula function is the Gaussian, or bivariate normal, copula function. Although Copula functions can be easily extended to the multivariate case, only the bivariate copula functions are considered for pairwise comparison and ranking methodology. The Gaussian copula function is of the form:

$$C(u,v) = \Phi_\rho\left(\Phi^{-1}(u), \Phi^{-1}(v)\right) \qquad 6\text{-}8$$

Given that $-1 \le \rho \le 1$ and where Φ_ρ is the joint Gaussian distribution function with correlation coefficient ρ, Φ^{-1} is the inverse of the Gaussian distribution function. The Gaussian copula function is not considered suitable for representing heavy tails. A limiting characteristic of the Gaussian copula is that it does not exhibit tail dependence. Copulas that do exhibit tail dependence are, e.g., the Student's t copula, Gumbel and Taylor Copula functions.

The bivariate Student's t-copula is a symmetric copula function that exhibits extreme tail dependence in both the upper and the lower tail. It is defined by

$$C(u,v) = t_{\rho,\eta}\left(t_\eta^{-1}(u), t_\eta^{-1}(v)\right) \qquad \text{6-9}$$

where $-1 \leq \rho \leq 1$. Kole et al. (2006) investigates the probability of joint extreme downward movements of stock market prices and conclude that the Gaussian copula underestimates, while the Gumbel copula overestimates this risk and concludes that there is evidence to support the Student's t copula function.

Notice that both the Gaussian and Student-t Copula functions are symmetrical and produce local tail dependences in both the upper and lower part of the tails of the multivariate distribution. Since volatility estimates exhibit only positive values by default, only the upper tail dependence is of interest. Two examples of asymmetric copula functions are the Gumbel and Clayton copulas.

The Gumbel-Hougaard or Gumbel copula function is an upper tail dependence measure representing extreme value distributions given by

$$C(u,v) = \exp\left\{-\left[-\ln(u)^{\delta} + \left(-\ln(v)^{\delta}\right)\right]^{\frac{1}{\delta}}\right\} \qquad \text{6-10}$$

with parameter $1 \leq \delta \leq \infty$. The parameter δ controls the strength of dependence, i.e. upper tail dependence is a function of Gumbel copula parameter $\lambda_u = 2 - 2^{\frac{1}{\delta}}$. When $\delta = 1$, there is no dependence ($\lambda_u = 0$) and when $\delta = +\infty$, there is perfect dependence ($\lambda_u = 1$).

The Clayton copula function is a lower tail dependence measure representing extreme value distribution given by

$$C(u,v) = \max\left\{u^{-\delta} + v^{-\delta} - 1, 0\right\}^{-1/\delta} \qquad \text{6-11}$$

Instead of considering the distribution of u and v, we can also consider the distribution of $1-u$ and $1-v$, which is also known as the rotated Copula. This approach only makes sense for asymmetric copula functions like the Gumbel or Clayton copulas. Since the Clayton copula returns only lower tail dependence, the rotated Clayton copula function would return the upper tail dependence, which is the point of our interest.

Since range-based volatility estimators, as well as realized volatility estimators, are considered to be heavy tailed on the right side of the distribution we find that the Gumbel copula function is adequate for measuring extreme dependences as it only exhibits upper tail dependences (Gumbel, 1960). Besides this the Gumbel copula is more asymmetric then the rotated Clayton. The tail dependence coefficients based on the rotated Clayton copula are used for robustness of the results.

The Gumbel and rotated Clayton copula functions are fitted to each bivariate distribution, i.e. each of the univariate pairs of range-based estimators. Each range-based volatility estimator is fitted against the $TTSE_t$. The empirical analysis includes the entire history of the selected emerging market indices.

The upper tail dependences are fitted with the Gumbel and rotated Clayton copulas using empirical CDF. Both copulas are fitted to each bivariate distribution, i.e. 12 univariate pairs. For each country under consideration, 12 tail dependences are estimated for each copula function.

7 Results of the Ranking Analysis

In the past decennia we have evidenced a vast increase of realized volatility models and ranking methodologies in the literature. The literature has not been unanimous on the volatility benchmark model for comparison of the results. In many cases the literature would claim the standard efficiency coefficient, which compares the competing estimator against the close-to-close estimator, as the ultimate efficiency measure. It is obvious that there can be major differences between volatility estimators due to their characteristics and specificities. One of the first to observe significant differences between range-based volatility estimators are Duque and Paxson (1997) who conclude that the choice of the range based volatility estimator is important for the significance of the estimator, which in their analysis was measured with the coefficient of efficiency. However, besides the fact that the choice of the estimator is of crucial importance, it is important to notice that the choice of the volatility benchmark model and the accompanying choice of the ranking methodology are perhaps even more important.

This chapter presents the results for each index based on various ranking methodologies. The first section discusses the results of the efficiency coefficient versus the close-to-close estimator as a widely used benchmark. Since the TTSE is a robust and unbiased benchmark of the integrated volatility, the coefficient of efficiency is adapted and compared against the TTSE in the second step. Next, various efficiency gain functions are discussed in sections 7.2 through 7.5. The Mincer-Zarnowitz regression, the loss function approach and the linear correlation results are common efficiency gain functions that can be found in the literature. The tail dependence approach is proposed as a fifth measure, which gives more information on the correlation of extreme movements that can only be found in the tails of the distributions. In section 7.6 the ranking results are summarized for conclusion. The final section drafts a conclusion based on the ranking analysis.

7.1 Results based on the Coefficient of efficiency approach

The coefficient of efficiency measures the relative variance of an alternative volatility estimator against the variance of the benchmark volatility estimator. A widely used benchmark for the efficiency ratio is the daily squared return, i.e. the close-to-close volatility estimator. This benchmark model is fairly easy to implement, but has the major disadvantage of not being a robust and unbiased estimator of the integrated variance. A robust and unbiased estimator of the integrated variance, also known as the ‘true’ variance, is the TTSE model. This model is also used as the benchmark volatility model throughout of this Thesis. The efficiency coefficient analysis is performed with two different volatility benchmark models and is shown in Table 7-1.

Table 7-1 shows the coefficient of efficiency for .BETI and .BUX. The benchmark model on the left panel is the close-to-close estimator and on the right panel it is the TTSE estimator.

.BETI		CC			TTSE		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
Excluded	Daily	1.00	1.00	1.00	0.54	0.55	0.32
	CO	1.00	1.00	1.00	0.54	0.55	0.32
	COC	1.00	1.00	1.00	0.54	0.55	0.32
	HL	0.71	0.73	0.57	0.38	0.40	0.19
	Park	1.97	2.01	1.59	1.07	1.10	0.51
	RS	1.55	1.53	1.78	0.84	0.83	0.57
	GK	**2.27**	**2.32**	**1.83**	**1.23**	**1.26**	**0.59**
included	HL	0.71	0.73	0.57	0.38	0.40	0.19
	Park*	1.97	2.01	1.59	1.07	1.10	0.51
	RS*	1.55	1.53	1.78	0.84	0.83	0.57
	GK*	2.27	2.32	1.83	1.23	1.26	0.59
	YZ	0.68	0.68	0.69	0.37	0.37	0.22
	TTSE	1.85	1.83	3.10	1.00	1.00	1.00

.BUX		CC			TTSE		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
excluded	Daily	1.00	1.00	1.00	0.33	0.33	0.31
	CO	1.40	1.51	1.02	0.47	0.50	0.32
	COC	1.20	1.25	1.02	0.40	0.41	0.32
	HL	0.75	0.77	0.67	0.25	0.26	0.21
	Park	2.08	2.14	1.86	0.70	0.71	0.58
	RS	2.01	1.98	**2.31**	0.67	0.66	**0.72**
	GK	**2.17**	**2.17**	2.28	**0.73**	**0.72**	0.71
included	HL	0.70	0.71	0.67	0.23	0.24	0.21
	Park*	1.60	1.60	1.84	0.54	0.53	0.58
	RS*	1.59	1.56	2.27	0.53	0.52	0.71
	GK*	1.66	1.64	2.25	0.56	0.54	0.70
	YZ	0.91	0.94	0.84	0.30	0.31	0.26

TTSE	2.99	3.02	3.20	1.00	1.00	1.00

In the first step the daily squared return (or the close-to-close volatility estimator) is used as the benchmark model. Secondly, the analysis is repeated with the TTSE. The efficiency coefficient is calculated for each of the 6 indices. The panel denoted with “CC” shows the results when the close-to-close estimator is chosen as the benchmark and the panel denoted with “TTSE” shows the results when the TTSE is chosen as the benchmark for the efficiency coefficient analysis. The bolded numbers in figures 7-1 through 7-3 indicate the maximum coefficient of efficiency for each of the 6 indices for each of the 3 different periods of time and 2 different benchmarks. The benchmark estimators have an efficiency of 1 by default and have therefore been excluded from the ranking analysis.

The coefficient of efficiency either increases or decreases with alternative volatility estimators. The coefficient of efficiencies shows that the choice of the volatility benchmark does not influence the ranking order of the volatility estimators. For example, the bolded volatility estimators for the .BUX index have the highest efficiency coefficient.

Based on the close-to-close benchmark the best volatility estimator is the Garman and Klass across all 3 periods of time. We exclude the TTSE from this analysis as the TTSE is a benchmark model. The second best volatility estimator is Parkinson and the third best is Roger and Satchel. The same ranking result holds for the case when the TTSE is chosen as the benchmark model. Although this is an expected outcome since the efficiency coefficient only changes linearly by changing the benchmark, it is interesting to compare the levels of the efficiency coefficients based on the two benchmarks.
Clearly the efficiency of the alternative estimators is lower when the unbiased TTSE estimator is applied. This is somehow also an expected outcome as the daily squared return is a biased benchmark of the integrated volatility.

Note that the intraday price observations in .BETI (table 7-1) and .SOFIX (table 7-2) did not include overnight returns in the data base. Therefore the results of the non-extended estimators are equal to the results that are based on the extended estimators.

Table 7-2 Results of the coefficient of efficiency for .CRBX and .PX. The benchmark model on the left panel is the close-to-close estimator and on the right panel it is the TTSE estimator.

.CRBX		CC			TTSE		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
excluded	Daily	1.00	1.00	1.00	0.80	0.80	0.51
	CO	0.98	0.98	1.04	0.78	0.78	0.52
	COC	0.81	0.81	1.03	0.65	0.65	0.52
	HL	0.76	0.76	0.71	0.60	0.62	0.36
	Park	2.10	2.12	1.96	1.68	1.71	0.99
	RS	2.00	2.00	2.43	1.60	1.61	1.22
	GK	**2.76**	**2.80**	**2.47**	**2.20**	**2.25**	**1.24**
included	HL	0.65	0.65	0.71	0.52	0.53	0.36
	Park*	1.31	1.30	1.95	1.05	1.05	0.98
	RS*	1.35	1.34	2.40	1.08	1.08	1.21
	GK*	1.49	1.48	2.44	1.19	1.19	1.23
	YZ	1.00	1.02	0.97	0.80	0.82	0.49
	TTSE	1.25	1.24	1.99	1.00	1.00	1.00
.PX		**CC**			**TTSE**		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
excluded	Daily	1.00	1.00	1.00	0.55	0.60	0.23
	CO	1.43	1.59	1.06	0.79	0.95	0.24
	COC	1.22	1.28	1.05	0.68	0.77	0.24
	HL	0.76	0.75	0.77	0.42	0.45	0.17
	Park	2.11	2.09	2.13	1.16	1.25	0.48
	RS	1.50	1.39	2.64	0.83	0.83	0.60
	GK	2.02	1.90	2.76	1.12	1.14	0.62
included	HL	0.71	0.69	0.77	0.39	0.42	0.17
	Park*	1.62	1.55	2.12	0.90	0.93	0.48
	RS*	1.31	1.21	2.64	0.72	0.72	0.60
	GK*	1.60	1.49	2.74	0.88	0.89	0.62
	YZ	**2.24**	**2.09**	**5.46**	**1.24**	**1.25**	**1.24**
	TTSE	1.81	1.67	4.42	1.00	1.00	1.00

Across the different indices we find that in most cases the Garman-Klass or the Yang-Zhang model show the maximum gain in efficiency. For the indices .PX and .SOFIX it is beneficial to include the overnight returns by applying the Yang-Zhang estimator as it increases the coefficient of efficiency. All other indices do not require overnight returns to reach maximum efficiency with the Garman-Klass estimator. One exception being .BUX during the second half of the empirical study where the Roger-Satchell model results in a higher efficiency gain than the Garman-Klass.

Table 7-3 Results of the coefficient of efficiency for .SOFIX and .WIG. The benchmark model on the left panel is the close-to-close estimator and on the right panel it is the TTSE estimator.

.SOFIX		CC			TTSE		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
excluded	Daily	1.00	1.00	1.00	0.84	0.55	0.99
	CO	1.00	1.00	1.00	0.84	0.55	0.99
	COC	1.00	1.00	1.00	0.84	0.55	0.99
	HL	0.54	0.60	0.48	0.45	0.33	0.50
	Park	1.49	1.68	1.34	1.24	0.92	1.40
	RS	1.09	1.51	0.88	0.91	0.83	0.95
	GK	1.48	1.82	1.26	1.24	1.00	1.34
included	HL	0.54	0.60	0.48	0.45	0.33	0.50
	Park*	1.49	1.67	1.34	1.24	0.92	1.40
	RS*	1.09	1.51	0.88	0.91	0.83	0.95
	GK*	1.48	1.82	1.26	1.24	1.00	1.34
	YZ	**2.37**	**3.09**	**1.97**	**1.98**	**1.70**	**2.11**
	TTSE	1.20	1.82	0.94	1.00	1.00	1.00
.WIG		CC			TTSE		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
excluded	Daily	1.00	1.00	1.00	0.40	0.43	0.30
	CO	1.30	1.24	1.47	0.52	0.53	0.44
	COC	1.11	1.05	1.31	0.44	0.45	0.39
	HL	0.65	0.60	0.89	0.26	0.26	0.27
	Park	1.81	1.66	2.48	0.72	0.71	0.75
	RS	1.72	1.54	2.69	0.68	0.66	0.81
	GK	**1.86**	**1.67**	**2.80**	**0.74**	**0.72**	**0.84**
included	HL	0.61	0.56	0.82	0.24	0.24	0.25
	Park*	1.39	1.28	1.87	0.55	0.55	0.56
	RS*	1.35	1.23	1.96	0.54	0.53	0.59
	GK*	1.42	1.30	2.01	0.56	0.56	0.61
	YZ	0.70	0.63	1.01	0.28	0.27	0.31
	TTSE	2.51	2.32	3.32	1.00	1.00	1.00

Only .BUX and .WIG show efficiency coefficients lower than 1 when the TTSE benchmark is used instead of the close-to-close benchmark model. This means that the efficiency coefficient does not indicate an increase in efficiency when alternative estimators are used instead of the benchmark. Another general observation is that when the TTSE benchmark is used instead the efficiency coefficient becomes lower compared to the case when the close-to-close benchmark is used. Compared to the efficiency coefficients found in the literature, a review based on the daily squared return can be found in table 3-2, we find much lower efficiency coefficients for the relevant estimators. This indicates that the efficiency coefficients clearly depend on the index and the chosen benchmark. Finally, we conclude that in almost all cases the efficiency coefficient did not show major variety between different periods of time. The advantage of the efficiency coefficient based on the daily squared return is that the ranking results will be comparable to the case when the TTSE is used and, based on the 3 periods of time, the results show to be robust. The results are in favour of the first hypothesis which states that range-based volatility estimators are appropriate models to estimate the 'true' volatility of stock indices. Across all indices and time periods we find evidence of range-based volatility estimators that support this hypothesis. In the alternative case either the daily squared return or the open-to-close estimator would show a better efficiency coefficient. However we find poor support for the alternative hypothesis, H.1.1, which states that range-based volatility estimators are different from each other, as the results show stability across time periods.

7.2 Results based on the Mincer Zarnowitz Regression analysis

Tables 7-4 and 7-5 show the coefficient of determination resulting from the Mincer-Zarnowitz regression for each of the indices. The analysis is performed for three different periods. The analysis of the regression analysis includes estimates based on daily horizon and estimates based on a horizon of 10 trading days. The reason for the additional analysis is to investigate the robustness of the results based on different time periods and a different estimation horizon. Appendix 3 can be consulted for more detailed information on the estimated parameters and the standard errors.

In general, the coefficients of determination based on a 10-day horizon exceeds the coefficients of determination based on a 1-day horizon. This indicates that there is a clear benefit in using a longer horizon for estimating the realized volatility. In most cases different sub periods also result in different optimal estimators. In most cases the lowest results are noted in the second sub-period, while the highest results are noted in the first sub-period. Since the recorded intraday data for .BETI and .SOFIX didn't include any overnight price movements, the extended versions of the range-based estimators show exactly identical results.

Based on the results of the total period, Parkinson results in the highest determination coefficient (0.62) for .Beti, Garman-Klass (0.57) for .BUX, High-Low (0.41) for .CRBX, Garman-Klass extended (0.56) for .PX, and Garman-Klass (0.55 and 0.59) for .SOFIX and

.WIG20, respectively. In case of the .WIG index the Garman-Klass estimator shows the highest coefficient of determination for all sub-periods and estimation horizons.

Table 7-4: The coefficient of determination, R^2, for the .BETI, .BUX and .CRBX indices of the encompassing Mincer-Zarnowitz regression on the daily realized volatility proxies. The (*) denotes the extended version of the volatility estimators that includes overnight jumps.

Indices		1 day horizon			10 day horizon		
.BETI	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
O/N jumps excluded	Daily	0.2107	0.2106	0.1398	0.6418	0.6846	0.3238
	CO	0.2107	0.2106	0.1398	0.6418	0.6846	0.3238
	COC	0.2107	0.2106	0.1398	0.6418	0.6846	0.3238
	HL	0.6213	0.6837	0.3681	0.8270	0.9255	0.4418
	Park	**0.6213**	0.6837	**0.3681**	**0.8270**	**0.9255**	**0.4418**
	RS	0.2929	0.3206	0.1477	0.6500	0.7393	0.2803
	GK	0.6011	**0.6881**	0.3146	0.7974	0.9178	0.3916
O/N jumps included	HL*	0.6213	0.6837	0.3681	0.8270	0.9255	0.4418
	Park*	0.6213	0.6837	0.3681	0.8270	0.9255	0.4418
	RS*	0.2929	0.3206	0.1477	0.6500	0.7393	0.2803
	GK*	0.6011	0.6881	0.3146	0.7974	0.9178	0.3916
	YZ	0.2132	0.2015	0.0233	0.5853	0.6214	0.1534
.BUX							
O/N jumps excluded	Daily	0.0705	0.0526	0.0903	0.4760	0.4244	0.4546
	CO	0.0906	0.0825	0.1061	0.5093	0.5647	0.4564
	COC	0.1218	0.0981	0.1071	0.6311	0.6199	0.4747
	HL	0.5219	0.5346	**0.4359**	0.8680	0.9080	0.7432
	Park	0.5219	0.5346	0.4359	0.8680	0.9080	0.7432
	RS	0.3866	0.4439	0.1877	0.8332	0.8698	0.5888
	GK	**0.5658**	**0.5852**	0.4309	0.8901	**0.9185**	0.7292
O/N jumps included	HL*	0.5099	0.4935	0.4348	**0.8906**	0.9047	**0.7538**
	Park*	0.4240	0.3682	0.4115	0.8446	0.8296	0.7284
	RS*	0.3585	0.3665	0.1762	0.8125	0.8323	0.5342
	GK*	0.4488	0.4067	0.3929	0.8442	0.8452	0.6766
	YZ	0.1624	0.1571	0.0247	0.5917	0.5970	0.2098
.CRBX							
O/N jumps excluded	Daily	0.1386	0.1431	0.0547	0.4316	0.3839	0.2142
	CO	0.1179	0.1149	0.0514	0.5649	0.5553	0.2040
	COC	0.0980	0.0885	0.0535	0.5624	0.5407	0.2138
	HL	**0.4097**	**0.3853**	0.3199	**0.7930**	0.7997	0.4121
	Park	0.4097	0.3853	0.3199	0.7930	**0.7997**	0.4121
	RS	0.1977	0.1674	0.1804	0.5213	0.4376	0.4403
	GK	0.3758	0.3387	0.3460	0.7141	0.6737	0.4786
O/N jumps included	HL*	0.3475	0.3089	0.3237	0.7609	0.7456	0.4190
	Park*	0.2616	0.2148	0.3204	0.6620	0.6162	0.4240
	RS*	0.1294	0.0960	0.1949	0.4302	0.3526	0.4393
	GK*	0.2187	0.1675	**0.3470**	0.5596	0.4877	**0.4833**
	YZ	0.0484	0.0282	0.0150	0.4408	0.3632	0.2174

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Table 7-5 shows the coefficient of determination, R^2, for the .PX, .SOFIX and .WIG indices of the encompassing Mincer-Zarnowitz regression on the daily realized volatility proxies. The * denotes the extended version of the volatility estimators that includes overnight jumps.

Indices		1 day horizon			10 day horizon		
.PX	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
O/N jumps excluded	Daily	0.1375	0.1737	0.0774	0.5089	0.6002	0.3849
	CO	0.0382	0.0417	0.0735	0.2877	0.4570	0.3749
	COC	0.1599	0.2183	0.0746	0.5688	0.7187	0.3763
	HL	0.2353	0.2885	0.2880	0.5213	0.7348	0.6009
	Park	0.2353	0.2885	0.2880	0.5213	0.7348	0.6009
	RS	0.1311	0.1836	0.0943	0.4338	0.5942	0.4229
	GK	0.2453	0.3037	0.2688	0.5148	0.7014	0.5966
O/N jumps included	HL*	0.4055	0.5069	0.2887	0.6894	0.8494	0.6015
	Park*	0.5315	0.6197	**0.2891**	0.8298	**0.9081**	**0.6023**
	RS*	0.4261	0.5484	0.1022	0.7910	0.8520	0.4295
	GK*	**0.5612**	**0.6431**	0.2725	**0.8463**	0.8992	0.5999
	YZ	0.1504	0.1449	0.0336	0.5570	0.5642	0.2655
.SOFIX							
O/N jumps excluded	Daily	0.1211	0.1370	0.1106	0.4202	0.3878	0.5203
	CO	0.1211	0.1370	0.1106	0.4202	0.3878	0.5203
	COC	0.1211	0.1370	0.1106	0.4202	0.3878	0.5203
	HL	0.5460	**0.5188**	0.5844	0.7312	**0.6774**	0.8238
	Park	0.5460	**0.5188**	0.5844	0.7312	**0.6774**	0.8238
	RS	0.3583	0.2987	0.4160	0.6904	0.5928	0.7759
	GK	**0.5498**	0.5146	**0.5885**	**0.7407**	0.6755	**0.8248**
O/N jumps included	HL*	0.5460	0.5188	0.5844	0.7312	0.6774	0.8238
	Park*	0.5460	0.5188	0.5844	0.7312	0.6774	0.8238
	RS*	0.3583	0.2987	0.4160	0.6904	0.5928	0.7759
	GK*	0.5498	0.5146	0.5885	0.7407	0.6755	0.8248
	YZ	0.0598	0.0390	0.0760	0.4070	0.2676	0.5292
.WIG							
O/N jumps excluded	Daily	0.0542	0.0593	0.0427	0.0545	0.0596	0.0427
	CO	0.1058	0.1140	0.0911	0.1065	0.1151	0.0911
	COC	0.1171	0.1304	0.0807	0.1180	0.1319	0.0807
	HL	0.5603	0.6125	0.4490	0.5615	0.6143	0.4490
	Park	0.5603	0.6125	0.4490	0.5615	0.6143	0.4490
	RS	0.4097	0.4825	0.2630	0.4158	0.4924	0.2630
	GK	**0.5933**	**0.6548**	**0.4578**	**0.5938**	**0.6554**	**0.4578**
O/N jumps included	HL*	0.5114	0.5633	0.3893	0.5125	0.5650	0.3893
	Park*	0.3738	0.4126	0.2681	0.3757	0.4155	0.2681
	RS*	0.3211	0.3866	0.1771	0.3228	0.3893	0.1771
	GK*	0.3836	0.4342	0.2559	0.3855	0.4372	0.2559
	YZ	0.1026	0.1383	0.0355	0.1033	0.1395	0.0355

7.3 Results based on the loss function approach

The MSE and QLike metrics have been calculated for each of the 6 indices and for 3 different time periods. The results of the MSE loss function are presented in tables 7-6 through 7-8. The results of the QLike loss function are presented in tables 7-9 through 7-11. For each time period the lowest MSE and QLike values are bolded. Table 7-6 shows the results of the loss function approach for the .BETI and .BUX indices.

Table 7-6 Results of the MSE analysis for .BETI and .BUX with different periods.

		.BETI			.BUX		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
O/N jumps excluded	Daily	1.133%	1.725%	0.540%	1.593%	2.228%	0.946%
	CO	1.133%	1.725%	0.540%	1.145%	1.411%	0.874%
	COC	1.133%	1.725%	0.540%	1.229%	1.597%	0.854%
	HL	1.315%	1.637%	0.990%	2.202%	2.691%	1.703%
	Park	**0.226%**	**0.301%**	0.150%	0.229%	0.276%	0.181%
	RS	0.532%	0.818%	0.244%	0.315%	0.368%	0.261%
	GK	0.248%	0.353%	**0.142%**	**0.182%**	**0.227%**	**0.135%**
O/N jumps included	HL	1.315%	1.637%	0.990%	2.642%	3.497%	1.769%
	Park*	0.226%	0.301%	0.150%	0.467%	0.722%	0.207%
	RS*	0.532%	0.818%	0.244%	0.499%	0.716%	0.276%
	GK*	0.248%	0.353%	0.142%	0.404%	0.642%	0.161%
	YZ	0.541%	0.832%	0.250%	0.556%	0.825%	0.282%

Table 7-7 shows the results of the loss function approach for the .CRBX and .PX indices.

Table 7-7 Results of the MSE analysis for .CRBX and .PX with different periods

		.CRBX			.PX		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
O/N jumps excluded	Daily	0.410%	0.587%	0.231%	0.919%	1.074%	0.762%
	CO	0.448%	0.665%	0.229%	0.928%	1.135%	0.719%
	COC	0.508%	0.793%	0.220%	0.734%	0.757%	0.711%
	HL	0.472%	0.715%	0.225%	1.225%	1.126%	1.323%
	Park	**0.140%**	**0.225%**	0.054%	0.288%	0.408%	0.167%
	RS	0.233%	0.388%	0.075%	0.417%	0.621%	0.213%
	GK	0.153%	0.255%	**0.051%**	0.275%	0.428%	0.121%
O/N jumps included	HL	0.550%	0.865%	0.228%	1.289%	1.246%	1.332%
	Park*	0.208%	0.361%	0.053%	0.183%	0.197%	0.169%
	RS*	0.297%	0.519%	0.071%	0.234%	0.262%	0.206%
	GK*	0.220%	0.388%	0.049%	**0.148%**	**0.174%**	**0.120%**
	YZ	0.305%	0.528%	0.079%	0.353%	0.541%	0.164%

The MSE results show that only the Parkinson and the Garman and Klass models have the lowest values across the different indices and time periods. Only for the .PX index we find evidence that overnight returns have and added value to the Garman and Klass estimator. The Garman and Klass estimator excluding overnight returns is supported with the lowest MSE for .BUX and .WIG. Parkinson has the lowest MSE for .SOFIX, .BUX and .CRBX. Only in the second period we find that the Garman and Klass model has the lowest MSE for .BUX and .CRBX. Table 7-8 shows the results of the loss function approach for the .CRBX and .PX indices.

Table 7-8 Results of the MSE analysis for .SOFIX and .WIG20 with different periods.

		.SOFIX			.WIG20		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
O/N jumps excluded	Daily	0.718%	0.758%	0.680%	1.303%	1.512%	1.090%
	CO	0.718%	0.758%	0.680%	0.882%	1.026%	0.735%
	COC	0.718%	0.758%	0.680%	0.951%	1.136%	0.762%
	HL	0.677%	0.843%	0.512%	1.486%	1.736%	1.231%
	Park	**0.173%**	**0.168%**	**0.178%**	0.155%	0.175%	0.134%
	RS	0.292%	0.301%	0.283%	0.231%	0.266%	0.195%
	GK	0.179%	0.170%	0.189%	**0.128%**	**0.145%**	**0.112%**
O/N jumps included	HL	0.677%	0.843%	0.512%	1.935%	2.355%	1.506%
	Park*	0.173%	0.168%	0.178%	0.394%	0.504%	0.281%
	RS*	0.292%	0.301%	0.283%	0.413%	0.520%	0.304%
	GK*	0.179%	0.170%	0.189%	0.355%	0.458%	0.251%
	YZ	0.390%	0.367%	0.413%	1.925%	2.450%	1.389%

Results based on the QLIKE loss function show similar outcome as the MSE loss function. Only the Parkinson and Garman and Klass model show lowest MSE for all indices across all periods. There is evidence to include overnight returns in the estimator in case of the .PX index. This is also supported for the second half of the investigated period in case of the .CRBX index. The Parkinson model shows the lowest MSE for both .BETI and .SOFIX across all investigated periods, while the Garman and Klass model shows the lowest MSE for .BUX and .WIG across all periods.

Table 7-9 Results of the QLIKE loss function for .BETI and .BUX with different periods.

		.BETI			.BUX		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
O/N jumps excluded	Daily	24.939%	23.996%	25.912%	120.611%	111.525%	129.841%
	CO	24.939%	23.996%	25.912%	136.873%	141.495%	132.110%
	COC	24.939%	23.996%	25.912%	63.524%	48.269%	79.099%
	HL	4.831%	3.622%	6.057%	38.495%	34.605%	42.528%
	Park	**1.925%**	**1.906%**	**1.960%**	7.562%	6.599%	8.559%
	RS	5.248%	5.928%	4.555%	14.849%	11.476%	18.265%
	GK	2.125%	2.241%	2.016%	**7.163%**	**6.344%**	**7.990%**
O/N jumps included	HL	4.831%	3.622%	6.057%	43.412%	42.451%	44.456%
	Park*	1.925%	1.906%	1.960%	10.475%	11.560%	9.384%
	RS*	5.248%	5.928%	4.555%	29.432%	16.098%	43.508%
	GK*	2.125%	2.241%	2.016%	9.739%	10.778%	8.671%
	YZ	2.733%	2.405%	3.064%	17.055%	18.801%	15.295%

Table 7-10 Results of the QLIKE loss function for .CRBX and .PX with different periods.

		.CRBX			.PX		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
O/N jumps excluded	Daily	27.158%	26.568%	27.803%	21.243%	18.891%	23.577%
	CO	26.589%	24.867%	28.408%	23.584%	23.356%	23.785%
	COC	20.514%	20.719%	20.396%	13.208%	8.355%	18.064%
	HL	3.828%	4.197%	3.452%	6.496%	4.785%	8.215%
	Park	**1.933%**	**2.014%**	1.858%	2.389%	2.984%	1.789%
	RS	3.976%	4.758%	3.188%	4.783%	5.384%	4.176%
	GK	2.000%	2.212%	1.789%	2.566%	3.522%	1.604%
O/N jumps included	HL	3.892%	4.277%	3.502%	6.759%	5.248%	8.277%
	Park*	1.883%	2.016%	1.757%	1.689%	**1.573%**	1.803%
	RS*	6.504%	9.121%	3.869%	5.353%	4.282%	6.422%
	GK*	1.958%	2.227%	**1.690%**	**1.638%**	1.679%	**1.596%**
	YZ	2.666%	3.300%	2.026%	2.730%	2.857%	2.600%

Table 7-11 Results of the QLIKE loss function for .SOFIX and .WIG with different periods.

		.SOFIX			.WIG20		
O/N jumps	Volatility estimator	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016	4-1-2010 1-4-2016	4-1-2010 15-2-2013	15-2-2013 1-4-2016
O/N jumps excluded	Daily	31.118%	32.318%	29.867%	26.008%	23.238%	28.848%
	CO	31.118%	32.318%	29.867%	26.474%	25.127%	27.838%
	COC	31.118%	32.318%	29.867%	9.823%	8.265%	11.428%
	HL	3.034%	3.422%	2.658%	5.711%	5.536%	5.892%
	Park	**2.292%**	**2.124%**	**2.465%**	1.201%	1.130%	1.273%
	RS	4.483%	3.975%	4.996%	2.553%	2.635%	2.465%
	GK	2.583%	2.363%	2.807%	**1.144%**	**1.080%**	**1.210%**
O/N jumps included	HL	3.034%	3.422%	2.658%	7.262%	7.333%	7.184%
	Park*	2.292%	2.124%	2.465%	2.125%	2.189%	2.056%
	RS*	4.483%	3.975%	4.996%	3.219%	3.134%	3.303%

GK*	2.583%	2.363%	2.807%	2.015%	2.088%	1.936%
YZ	3.185%	3.158%	3.210%	7.036%	7.023%	7.041%

7.4 Pearson Correlation Coefficients

The Pearson correlation coefficient is calculated for three different periods and for each of the range-based volatility estimates. Tables 7-12, 7-13 and 7-14 show the results for the .BETI, .BUX, .CRBX, .PX, .SOFIX and .WIG20 indices. Each table shows the results for three different periods, denoting the January 2010 to April 2016, January 2010 to February 2013 and February 2013 to April 2016.

Table 7-12 presents empirical results of the Pearson correlation function for .BETI and .BUS given three different periods.

		.BETI			.BUX		
O/N jumps	Volatility estimator	4/1/2010 1/4/2016	4/1/2010 15-2-2013	15-2-2013 1/4/2016	4/1/2010 1/4/2016	4/1/2010 15-2-2013	15-2-2013 1/4/2016
excluded	Daily	0.6240	0.6135	0.7765	0.4428	0.3853	0.7064
	CO	0.6240	0.6135	0.7765	0.5200	0.4767	0.7180
	COC	0.6240	0.6135	0.7765	0.5333	0.4826	0.7170
	HL	0.9124	0.9249	**0.8772**	0.8373	0.8279	**0.8770**
	Park	0.9124	0.9249	**0.8772**	0.8373	0.8279	**0.8770**
	RS	0.7879	0.7904	0.7914	0.8882	0.8988	0.7902
	GK	**0.9131**	**0.9283**	0.8513	**0.8998**	**0.9002**	0.8759
included	HL*	0.9124	0.9249	0.8772	0.8324	0.8178	0.8760
	Park*	0.9124	0.9249	0.8772	0.7619	0.7334	0.8687
	RS*	0.7879	0.7904	0.7914	0.8222	0.8208	0.7740
	GK*	0.9131	0.9283	0.8513	0.8122	0.7978	0.8601
	YZ	0.4351	0.4245	0.4016	0.5830	0.6099	0.3032

The upper panel of the tables show the results for the range-based volatility estimates that exclude overnight returns. The lower panel of the tables shows the results for the estimates that include overnight returns. The bolded figures indicate the highest correlation coefficient for each of the investigated time periods.

Note that the only difference between the Parkinson range-based volatility estimate (eq. 3-8) and the High-Low volatility estimate (eq. 3-9) is a constant factor in the Parkinson estimate. Therefore the Pearson correlation between these two estimates is expected to be equal.

Table 7-13 presents empirical results of the Pearson correlation function for .CRBX and .PX given three different periods.

		.CRBX			.PX		
O/N jumps	Volatility estimator	4/1/2010 1/4/2016	4/1/2010 15-2-2013	15-2-2013 1/4/2016	4/1/2010 1/4/2016	4/1/2010 15-2-2013	15-2-2013 1/4/2016
excluded	Daily	0.8080	0.8126	0.5160	0.5314	0.5384	0.5727
	CO	0.7750	0.7779	0.4981	0.3843	0.4045	0.5714
	COC	0.6466	0.6443	0.5019	0.5760	0.6054	0.5721
	HL	0.8013	0.8196	0.6708	0.7193	0.7506	0.7139
	Park	**0.8218**	**0.8239**	0.6708	0.7193	0.7506	0.7139
	RS	0.4221	0.4076	0.6596	0.7283	0.7427	0.5814
	GK	0.6529	0.6466	**0.7091**	0.7466	0.7620	0.7089
included	HL*	0.7076	0.7033	0.6735	0.8136	0.8446	0.7143
	Park*	0.5142	0.5041	0.6743	0.8802	0.8991	**0.7149**
	RS*	0.2864	0.2711	0.6622	0.8968	0.9088	0.5844
	GK*	0.3557	0.3408	0.7003	**0.9145**	**0.9249**	0.7109
	YZ	0.2518	0.2356	0.2206	0.5054	0.4855	0.3731

Table 7-14 presents empirical results of the Pearson correlation function for .SOFIX and .WIG20 given three different periods.

		.SOFIX			.WIG20		
O/N jumps	Volatility estimator	4/1/2010 1/4/2016	4/1/2010 15-2-2013	15-2-2013 1/4/2016	4/1/2010 1/4/2016	4/1/2010 15-2-2013	15-2-2013 1/4/2016
excluded	Daily	0.5471	0.5979	0.5624	0.5113	0.5194	0.4929
	CO	0.5471	0.5979	0.5624	0.6129	0.6101	0.6193
	COC	0.5471	0.5979	0.5624	0.6257	0.6244	0.6092
	HL	0.8251	**0.8180**	**0.8546**	0.8639	0.8740	0.8068
	Park	0.8251	**0.8180**	**0.8546**	0.8639	0.8740	0.8068
	RS	0.8094	0.7498	0.8308	0.8800	0.9039	0.7496
	GK	**0.8266**	0.8129	0.8428	**0.8901**	**0.9046**	**0.8125**
included	HL*	0.8251	0.8180	0.8546	0.8564	0.8688	0.7834
	Park*	0.8251	0.8180	0.8546	0.7916	0.8068	0.6972
	RS*	0.8094	0.7498	0.8308	0.8149	0.8458	0.6362
	GK*	0.8266	0.8129	0.8428	0.8088	0.8311	0.6763
	YZ	0.4879	0.3811	0.5297	0.5168	0.5340	0.3760

For .Beti and .SOFIX there were no overnight prices observed. Therefore the correlation of the extended estimates is equal to the correlation of the non-extended ones. This also holds for the close-to-open-to-close estimator, which would result in an equal correlation coefficient as the close-to-open estimator. The results show that in all cases either the Parkinson or the Garman and Klass estimates show the highest correlation with the TTSE benchmark. Only in the case of .PX we find evidence that overnight returns add value to the estimates. Analysing the results based on the entire period we find that the Garman and Klass estimates show the highest correlation coefficient in all cases except for .CRBX where the Parkinson model shows a better results. The second half of the investigated period shows that a change in range-based volatility model would result in a higher correlation in almost all cases except for .WIG.

7.5 Results based on the Tail Dependence approach

The results of the tail dependence estimates are presented in Tables 7-15 through 7-20. The left panel in each of the tables presents the tail dependence coefficients based on the Gumbel or Rotated Taylor Copula function. The panel on the right presents the rankings which are defined as follows: the estimator with the highest tail dependence coefficient gets rank value 13 (equals total number of estimators), while the estimator with the lowest tail dependence coefficient gets rank value 1. The tail dependence estimates are calculated for three different periods. Tables 7-15 and 7-16 present the tail dependence coefficients estimated for the maximum available empirical period from January 2010 to April 2016. Tables 7-17 and 7-18 present the first sub-period from January 2010 to February 2013 and tables 7-19 and 7-20 present the second sub-period denoting the period from February 2013 to April 2016. For each of the three periods the first table considers the range-based estimators that exclude overnight returns and the second table includes the overnight returns, either by default or with an add-on.

In general the results based on the entire empirical period, presented in the right panel of tables 7-15 and 7-16, show that the rankings of the tail dependence coefficients based on the Gumbel and rotated Clayton copula are rather comparable for most of the indices. The ranking results between the copula functions are similar except for the .SOFIX, where the Gumbel copula suggests the High-Low estimator, while the rotated Clayton suggests the Garman-Klass estimator. For all other indices the choice of the copula function would not change the choice of the estimator based on the highest ranking. Our choice for the tail dependence coefficient is based on the Gumbel copula function, whilst for the .SOFIX index we will take into account the result of the rotated Clayton copula.

Table 7-15 shows in the left panel the tail dependence coefficients based on the Gumbel and rotated Clayton copula (Clayton*) function for the range-based volatility estimators that exclude overnight jumps during the period January 2010 – April 2016. The panel on the right hand side shows the total ranking categorization based on the tail dependence coefficients shown in table 7-15 and 7-16.

1/4/2016 4/1/2010	Upper tail dependence coefficient							Ranking results						
Copula	SD	CC	CO	HL	Park	RS	GK	SD	CC	CO	HL	Park	RS	GK
.BETI														
Gumbel	0.53	0.48	0.48	**0.72**	**0.72**	0.53	0.71	7	4	4	**13**	**13**	6	9
Clayton*	0.56	0.53	0.53	**0.75**	**0.75**	0.59	0.74	5	4	4	**13**	**13**	7	9
.BUX														
Gumbel	0.40	0.30	0.42	0.67	0.67	0.50	0.65	3	1	4	12	12	6	10
Clayton*	0.38	0.31	0.46	0.71	0.71	0.58	0.70	3	1	4	12	12	7	10
CRBX														
Gumbel	0.27	0.36	0.33	0.63	0.63	0.58	0.64	1	3	2	9	9	6	10
Clayton*	0.23	0.38	0.34	0.66	0.66	0.63	0.68	1	3	2	9	9	6	11
.PX														
Gumbel	0.41	0.45	0.29	0.45	0.45	0.28	0.41	4	5	2	8	8	1	3
Clayton*	0.39	0.48	0.26	0.50	0.50	0.32	0.48	3	4	1	8	8	2	5
.SOFIX														
Gumbel	0.32	0.39	0.39	**0.73**	**0.73**	0.61	0.73	1	4	4	**13**	**13**	7	9
Clayton*	0.38	0.44	0.44	0.79	0.79	0.70	**0.79**	1	4	4	11	11	7	**13**
.WIG														
Gumbel	0.46	0.33	0.30	0.61	0.61	0.47	**0.63**	5	2	1	11	11	6	**13**
Clayton*	0.52	0.33	0.31	0.66	0.66	0.55	**0.69**	5	2	1	11	11	7	**13**
∑ rank Gumbel								**21**	**17**	**15**	**63**	**61**	**32**	**54**
∑ rank Clayton*								**18**	**16**	**14**	**61**	**59**	**36**	**61**

Amongst the estimators that exclude overnight jumps, presented in table 7-15, the High-Low estimator shows the highest total ranking value of 63 across the set of indices. The average tail dependence coefficient of the High-Low estimator across the indices is 0.64. The second best ranked estimator is the Parkinson volatility estimator with a total ranking value of 61 (a slightly lower average tail dependence coefficient rounded at 0.63) and the Garman-Klass with a total ranking value of 48 (average tail dependence coefficient of 0.63). The Standard Deviation, the Daily Close-to-Close and Close-to-Open estimators have by far the lowest total ranking results of 21, 17 and 15, respectively. They also have the lowest average tail dependence coefficient of 0.40, 0.38 and 0.37, respectively.

Amongst the estimators that include overnight returns, presented in table 7-13, the High-Low extended, Parkinson extended and Garman-Klass extended show the highest ranking values of 71, 66 and 60, respectively. Their average tail dependence coefficient is 0.66, 0.65 and 0.65, respectively. Surprisingly, the Close-to-Open-to-Close and the Yang-Zhang have the lowest overall ranking value amongst the estimators that include overnight returns (30 and 22, respectively) and the lowest average tail dependence coefficient of 0.44 and 0.44 respectively. The total ranking value of the extended Roger-Satchell estimator is 43 and has an average tail-dependence coefficient of 0.55.

Table 7-16 shows in the left panel the tail dependence coefficients based on the Gumbel and rotated Clayton copula (Clayton*) function for the range-based volatility estimators that include overnight jumps during the period January 2010 – April 2016. The panel on the right hand side shows the total ranking categorization based on the tail dependence coefficients shown in table 7-15 and 7-16.

1/4/2016 4/1/2010	Upper tail dependence coefficient						Ranking results					
Copula	COC	HL Ext	Park Ext	RS Ext	GK Ext	Yang Zhang	COC	HL Ext	Park Ext	RS Ext	GK Ext	Yang Zhang
.BETI												
Gumbel	0.48	**0.72**	**0.72**	0.53	0.71	0.47	4	**13**	**13**	6	9	1
Clayton*	0.53	**0.75**	**0.75**	0.59	0.74	0.47	4	**13**	**13**	7	9	1
.BUX												
Gumbel	0.47	**0.67**	0.62	0.53	0.61	0.36	5	**13**	9	7	8	2
Clayton*	0.52	**0.72**	0.66	0.58	0.64	0.36	5	**13**	9	6	8	2
CRBX												
Gumbel	0.37	0.64	0.65	0.60	**0.66**	0.46	4	11	12	7	**13**	5
Clayton*	0.38	0.67	0.69	0.65	**0.70**	0.46	4	10	12	7	**13**	5
.PX												
Gumbel	0.54	0.60	**0.66**	0.52	0.62	0.45	10	11	**13**	9	12	6
Clayton*	0.58	0.67	**0.73**	0.60	0.69	0.49	9	11	**13**	10	12	6
.SOFIX												
Gumbel	0.39	**0.73**	**0.73**	0.61	0.73	0.56	4	13	13	7	9	5
Clayton*	0.44	0.79	0.79	0.70	**0.79**	0.66	4	11	11	7	13	5
.WIG												
Gumbel	0.41	0.62	0.54	0.49	0.55	0.37	4	12	8	7	9	3
Clayton*	0.44	0.68	0.59	0.53	0.60	0.36	4	12	8	6	9	3
∑ rank Gumbel							31	71	66	43	60	22
∑ rank Clayton*							30	68	64	43	64	22

When tail dependence becomes important for the individual markets we find that the Parkinson estimators is the best fit for the .BETI index, Garman-Klass for the WIG20., the High-Low or the Garman-Klass for .SOFIX, High-Low extended for .BUX, Parkinson extended for .BETI and .PX, and Garman-Klass Extended for .CRBX. Since the recorded intraday data for both, .BETI and .SOFIX, did not include any overnight changes, there are also no differences between the results of the estimators that include or exclude overnight returns.

The overall conclusion that can be drawn, based on the total empirical analysis of the total ranking values is that the extended range-based volatility estimates have a higher tail dependence coefficient compared to the estimates that do not include overnight returns.

Table 7-17 shows in the left panel the tail dependence coefficients based on the Gumbel and rotated Clayton copula (Clayton*) function for the range-based volatility estimators that exclude overnight jumps during the period January 2010 – February 2013. The panel on the right hand side shows the total ranking categorization based on the tail dependence coefficients shown in table 7-17 and 7-18.

4/1/2010 15/2/2013	Upper tail dependence coefficient							Ranking results						
Copula	SD	CC	CO	HL	Park	RS	GK	SD	CC	CO	HL	Park	RS	GK
.BETI														
Gumbel	0.49	0.50	0.50	**0.72**	**0.72**	0.53	0.71	2	5	5	**13**	**13**	7	9
Clayton*	0.52	0.56	0.56	**0.77**	**0.77**	0.59	0.75	2	5	5	**13**	**13**	7	9
.BUX														
Gumbel	0.33	0.31	0.43	**0.69**	**0.69**	0.51	0.67	3	1	4	**13**	**13**	7	10
Clayton*	0.29	0.35	0.47	**0.74**	**0.74**	0.61	0.73	1	3	4	**13**	**13**	7	10
CRBX														
Gumbel	0.15	0.38	0.36	0.64	0.64	0.59	0.65	1	3	2	8	9	6	11
Clayton*	0.09	0.42	0.39	0.69	0.69	0.65	0.70	1	4	2	9	8	6	11
.PX														
Gumbel	0.41	0.48	0.33	0.55	0.55	0.37	0.51	3	5	1	8	8	2	6
Clayton*	0.39	0.53	0.31	0.61	0.61	0.46	0.59	2	5	1	8	8	3	6
.SOFIX														
Gumbel	0.42	0.23	0.23	0.49	0.49	0.51	**0.54**	4	3	3	9	9	11	**13**
Clayton*	0.54	0.23	0.23	0.59	0.59	0.61	**0.63**	4	3	3	9	9	11	**13**
.WIG														
Gumbel	0.48	0.33	0.31	0.63	0.63	0.49	**0.65**	5	2	1	10	11	6	**13**
Clayton*	0.57	0.34	0.34	0.70	0.70	0.59	**0.72**	6	1	2	10	11	7	**13**
∑ rank Gumbel								**18**	**19**	**16**	**61**	**60**	**39**	**62**
∑ rank Clayton*								**16**	**21**	**17**	**60**	**61**	**41**	**62**

The results of the tail dependence analysis based on the first sub-period, January 2010 to February 2013, are presented in tables 7-15 and 7-16. The results of the tail dependence coefficient based on the first sub-period differ from the total period results.

Table 7-18 shows in the left panel the tail dependence coefficients based on the Gumbel and rotated Clayton copula (Clayton*) function for the range-based volatility estimators that include overnight jumps during the period January 2010 – February 2013. The panel on the right hand side shows the total ranking categorization based on the tail dependence coefficients shown in table 7-17 and 7-18.

4/1/2010 15/2/2013	Upper tail dependence coefficient						Ranking results					
Copula	COC	HL Ext	Park Ext	RS Ext	GK Ext	Yang Zhang	COC	HL Ext	Park Ext	RS Ext	GK Ext	Yang Zhang
.BETI												
Gumbel	0.50	0.72	0.72	0.53	0.71	0.43	5	13	13	7	9	1
Clayton*	0.56	0.77	0.77	0.59	0.75	0.43	5	13	13	7	9	1
.BUX												
Gumbel	0.45	0.68	0.62	0.50	0.59	0.32	5	11	9	6	8	2
Clayton*	0.51	0.74	0.67	0.57	0.64	0.33	5	11	9	6	8	2
CRBX												
Gumbel	0.39	0.65	0.66	0.61	**0.67**	0.41	4	10	12	7	**13**	5
Clayton*	0.43	0.70	0.71	0.67	**0.73**	0.41	5	10	12	7	**13**	3
.PX												
Gumbel	0.56	0.68	**0.70**	0.57	0.68	0.44	9	12	**13**	10	11	4
Clayton*	0.61	0.74	**0.77**	0.66	0.75	0.51	9	11	**13**	10	12	4
.SOFIX												
Gumbel	0.23	0.49	0.49	0.51	0.54	0.49	3	9	9	11	13	9
Clayton*	0.23	0.59	0.59	0.61	0.63	0.59	3	9	9	11	13	9
.WIG												
Gumbel	0.41	0.64	0.56	0.51	0.57	0.39	4	12	8	7	9	3
Clayton*	0.46	0.70	0.62	0.56	0.63	0.40	4	12	8	5	9	3
∑ rank Gumbel							**30**	**67**	**62**	**48**	**63**	**24**
∑ rank Clayton*							**31**	**64**	**64**	**46**	**64**	**22**

The results of the tail dependence analysis based on the second sub-period, February 2013 to April 2016, are presented in tables 7-16 and 7-17. The results of the tail dependence coefficient based on the first sub-period differ from the total period results.

Table 7-19 shows in the left panel the tail dependence coefficients based on the Gumbel and rotated Clayton copula (Clayton*) function for the range-based volatility estimators that exclude overnight jumps during the period February 2013 – April 2016. The panel on the right hand side shows the total ranking categorization based on the tail dependence coefficients shown in table 7-19 and 7-20.

15/2/2013 1/4/2016	Upper tail dependence coefficient							Ranking results						
Copula	SD	CC	CO	HL	Park	RS	GK	SD	CC	CO	HL	Park	RS	GK
.BETI														
Gumbel	0.12	0.35	0.35	0.55	0.55	0.56	**0.58**	1	5	5	9	9	11	**13**
Clayton*	0.00	0.30	0.30	0.57	0.57	0.61	**0.62**	1	5	5	9	9	11	**13**
.BUX														
Gumbel	0.34	0.38	0.40	0.59	0.59	0.49	0.56	1	3	4	11	12	7	9
Clayton*	0.34	0.43	0.48	0.67	0.67	0.62	0.66	1	3	4	11	12	8	10
CRBX														
Gumbel	0.33	0.30	0.29	0.55	0.55	0.46	0.56	4	3	1	8	9	6	12
Clayton*	0.28	0.35	0.30	0.58	0.58	0.50	0.58	1	5	3	9	10	6	8
.PX														
Gumbel	0.34	0.36	0.35	**0.61**	**0.61**	0.47	0.60	2	5	3	**13**	**13**	6	9
Clayton*	0.35	0.39	0.38	0.65	0.65	0.52	0.64	2	5	3	12	12	7	9
.SOFIX														
Gumbel	0.24	0.42	0.42	0.65	0.65	0.55	**0.65**	1	5	5	11	11	7	**13**
Clayton*	0.20	0.44	0.44	0.68	0.68	0.61	**0.69**	1	5	5	11	11	7	**13**
.WIG														
Gumbel	0.41	0.35	0.40	0.63	0.63	0.57	**0.66**	5	2	3	11	12	9	**13**
Clayton*	0.37	0.36	0.44	0.66	0.66	0.63	**0.71**	3	2	5	11	12	9	**13**
∑ rank Gumbel								**14**	**23**	**21**	**63**	**66**	**46**	**69**
∑ rank Clayton*								**9**	**25**	**25**	**63**	**66**	**50**	**64**

Table 7-20 shows in the left panel the tail dependence coefficients based on the Gumbel and rotated Clayton copula (Clayton*) function for the range-based volatility estimators that include overnight jumps during the period February 2013 – April 2016. The panel on the right hand side shows the total ranking categorization based on the tail dependence coefficients shown in table 7-19 and 7-20

15/2/2013 1/4/2016	Upper tail dependence coefficient						Ranking results					
Copula	COC	HL Ext	Park Ext	RS Ext	GK Ext	Yang Zhang	COC	HL Ext	Park Ext	RS Ext	GK Ext	Yang Zhang
.BETI												
Gumbel	0.35	0.55	0.55	0.56	0.58	0.26	5	9	9	11	13	2
Clayton*	0.30	0.57	0.57	0.61	0.62	0.19	5	9	9	11	13	2
.BUX												
Gumbel	0.47	**0.62**	0.58	0.48	0.54	0.35	5	**13**	10	6	8	2
Clayton*	0.54	**0.69**	0.64	0.56	0.60	0.40	5	**13**	9	6	7	2
CRBX												
Gumbel	0.29	0.56	0.56	0.47	**0.57**	0.34	2	10	11	7	**13**	5
Clayton*	0.31	0.58	**0.59**	0.51	0.59	0.29	4	11	**13**	7	12	2
.PX												
Gumbel	0.35	0.61	0.61	0.47	0.60	0.33	4	11	10	7	8	1
Clayton*	0.38	**0.65**	0.65	0.52	0.64	0.33	4	**13**	10	6	8	1
.SOFIX												
Gumbel	0.42	0.65	0.65	0.55	0.65	0.29	5	11	11	7	13	2
Clayton*	0.44	0.68	0.68	0.61	0.69	0.30	5	11	11	7	13	2
.WIG												
Gumbel	0.40	0.60	0.54	0.51	0.55	0.31	4	10	7	6	8	1
Clayton*	0.43	0.64	0.57	0.55	0.59	0.24	4	10	7	6	8	1
∑ rank Gumbel							**25**	**64**	**58**	**44**	**63**	**13**
∑ rank Clayton*							**27**	**67**	**59**	**45**	**59**	**10**

7.6 Summary and discussion of the efficiency gain results

This chapter presents an overview of the main results of the ranking analysis, which was provided in sections 7-1 through 7-6. For each of the ranking methodologies a summary is presented based on the benchmark of choice (if relevant), the ranking methodology and on the scope of the empirical data. The summary table only presents the volatility estimators that were previously bolded. These figures indicate the least biased estimators or the estimators that performed the closest to the benchmark. Tables 7-21 to 7-23 provide a summary of the results

Table 7-21 summarizes the results of the ranking analysis for each of the ranking methodologies for .BETI and .BUX.

Ranking methodology	.BETI 4/1/2010 1/4/2016	.BETI 4/1/2010 15/2/2013	.BETI 15/2/2013 1/4/2016	.BUX 4/1/2010 1/4/2016	.BUX 4/1/2010 15/2/2013	.BUX 15/2/2013 1/4/2016
Coefficient of efficiency						
Benchmark: CC	GK	GK	GK	GK	GK	RS
Benchmark: TTSE	GK	GK	GK	GK	GK	RS
Mince-Zarnowitz regression analysis						
Horizon: 1day	Park	GK	Park	GK	GK	HL
Horizon: 10 days	Park	Park	Park	HL*	GK	HL*
Loss function approach						
Loss function: MSE	Park	Park	GK	GK	GK	GK
Loss function: QLike	Park	Park	Park	GK	GK	GK
Pearson’s linear correlation function						
Correlation coefficient	GK	GK	HL/Park	GK	GK	HL/Park
Tail Correlation function						
Copula function: Gumbel	HL/Park	HL/Park	GK	HL*	HL	HL*
Copula function: Clayton	HL/Park	HL/Park	GK	HL*	HL	HL*

Table 7-22 summarizes the results of the ranking analysis for each of the ranking methodologies for .CRBX and .PX.

Ranking methodology	.CRBX 4/1/2010 1/4/2016	.CRBX 4/1/2010 15/2/2013	.CRBX 15/2/2013 1/4/2016	.PX 4/1/2010 1/4/2016	.PX 4/1/2010 15/2/2013	.PX 15/2/2013 1/4/2016
Coefficient of efficiency						
Benchmark: CC	GK	GK	GK	YZ	YZ	YZ
Benchmark: TTSE	GK	GK	GK	YZ	YZ	YZ
Mince-Zarnowitz regression analysis						
Horizon: 1day	HL	HL	GK*	GK*	GK*	Park*
Horizon: 10 days	HL	Park	GK*	GK*	Park*	Park*
Loss function approach						
Loss function: MSE	Park	Park	GK	GK*	GK*	GK*
Loss function: QLike	Park	Park	GK*	GK*	Park*	GK*
Pearson’s linear correlation function						
Correlation coefficient	Park	Park	GK	GK*	GK*	Park*
Tail Correlation function						

Copula function: Gumbel	GK*	GK*	GK*	Park*	Park*	HL/Park
Copula function: Clayton	GK*	GK*	Park*	Park*	Park*	HL*

Table 7-23 summarizes the results of the ranking analysis for each of the ranking methodologies for .SOFIX and .WIG.

	.SOFIX			.WIG		
Ranking methodology	4/1/2010 1/4/2016	4/1/2010 15/2/2013	15/2/2013 1/4/2016	4/1/2010 1/4/2016	4/1/2010 15/2/2013	15/2/2013 1/4/2016
Coefficient of efficiency						
Benchmark: CC	YZ	YZ	YZ	GK	GK	GK
Benchmark: TTSE	YZ	YZ	YZ	GK	GK	GK
Mince-Zarnowitz regression analysis						
Horizon: 1day	GK	HL/Park	GK	GK	GK	GK
Horizon: 10 days	GK	HL/Park	GK	GK	GK	GK
Loss function approach						
Loss function: MSE	Park	Park	Park	GK	GK	GK
Loss function: QLike	Park	Park	Park	GK	GK	GK
Pearson’s linear correlation function						
Correlation coefficient	GK	HL/Park	HL/Park	GK	GK	GK
Tail Correlation function						
Copula function: Gumbel	HL/Park	GK	GK	GK	GK	GK
Copula function: Clayton	GK	GK	GK	GK	GK	GK

The results show that in most cases there is no unanimous answer to the question which of the volatility estimators to use in which case. One exception is the .WIG index, where the Garman and Klass model outperforms all the other volatility estimators in all three time periods.

An overall conclusion that can be drawn from the results is that the the performance of the Parkinson and the Garman and Klass model, either with or without the extension to include overnight returns, outperform all the other models in most of the indices and the defined time periods.

This also provides answer to the first hypothesis, H.1, which states that Range-based volatility estimators are appropriate models to estimate the 'true' volatility of stock indices. Based on the analysis in which a wide set of range-based volatility estimators and a wide set of stock market indices are compared with various and divergent ranking methodologies against the 'true' volatility we come to the conclusion that range-based volatlility estimators, i.e. the Parkinson and the Garman and Klass models (either with or without the extension to include overnight returns), proof to estimate the 'true' volatility resaonably well. The evidence of this statement lays with the performance of the various ranking methodologies.

The coefficient of efficiency was performed with 2 different benchmark models. One benchmark model was based on the close-to-close estimator as was proposed in the literature, and another one was based on the TTSE model, which can be seen as an improvement to the previous literature that proposed or utilized the coefficient of efficiency to rank range-based volatility estimators. However, from the empirical analysis we find that the ranking results of the two efficiency coefficient methodologies are exactly the same and do not depend on the choice of the benchmark model. This conclusion also holds across different time periods where we notice that the coefficient of efficiency provides rather stable results. Based on the coefficient of efficiency we find that for .BETI, CRBX and .WIG the Garman and Klass model provides the best result across all three time periods. For .BUX we notice that during the second half of the period the Roger and Satchell model outperformed, while in all other cases Garman and Klass model outperformed. Only for .PX and .SOFIX the Yang and Zhang model outperformed for all three time periods.

The Mincer-zarnowitz regression was performed with a 1 and 10 day horizon and shows various ranking results. The only estimators that have outperformed in the ranking analysis are the Garman and Klass, Parkinson and the High-Low estimators. Only .BUX and .PX include an extension for overnight returns, while in all other cases the overnight returns didn't add value.

The Loss function approach was performed with two loss functions, the MSE and the MAE. Across all stock market indices and across all time periods only the Garman and Klass and the Parkinson estimators were outperforming in the ranking methodology. Including overnight returns was beneficial only in the case of .PX and for the second time period for .CRBX.

A similar conclusion can be drawn when basing the ranking methodology on the Pearson Correlation Coefficient. Only the Garman and Klass, the Parkinson and the High-Low estimators outperform. Including overnight returns was only beneficial for .PX. In all other cases the inclusion of overnight returns didn't proof to outperform the standard range-based volatility models, i.e. Garman and Klass and Parkinson.

The Tail correlation methodology indicates the performance of the estimators in the tail of the distribution. This is the only ranking methodology that focuses on the performance in the extremes. The results of the Tail correlation ranking methodology show the importance of including overnight returns in the volatility estimation models, which can be traced back in the results of .BUX, .CRBX and .PX. The extended models that include overnight returns have no impact on .BETI and .SOFIX as for these indices there were no records of overnight jumps registered. These two indices are therefore out of scope for the auxilliary hypothesis, H1.2., which states that the efficiency of classical range-based volatility estimators can be increased by including overnight returns. We find evidence in all cases, excluding .BETI and .SOFIX, except for .WIG. Please note that .WIG is the only stock market index for which we can conclude that the Garman and Klass model outperforms in all of the applied ranking methodologies. Based on the results we can conclude that in most cases the overnight returns increase the efficiency based on the Tail Correlation ranking methodology.

The second hypothesis, H.2., states that the dependence between the 'true' and the range-based volatility estimator is non-linear and shows dependence in the tails of the distirbutions. The positive tail correlations in tables 7-15 through 7-20 provide evidence in favor of this hypothesis. The tail correlation is indeed different from zero in almost all cases and the tail correlation of the best ranked range-based volatility estimators lays in the third quantile, between 0.5 and 0.75, in all the analysed cases.

8 APPLICATION I - VALUE-AT-RISK

8.1 Introduction

Value-at-Risk (VaR) is a risk metric that measures the future uncertainty of an investment by providing information on the downside risk in the form of an amount that could be lost with a chosen probability. An attractive feature of VaR is that it is applicable to all types of risk and, as a risk metric, it is easily comparable across different markets, exposures, but also across models. This is probably the reason why VaR has gained in popularity in the past years and has become a standard in both science as in the industry.

A vast amount of VaR models can be found in the literature of which the parametric linear VaR model is one of the simplest and most popular one. Consequently the use, but also in certain amounts the abuse, of VaR models has received many criticism. The liquidity crisis of 2007-2008 has confirmed that the critics weren't unjustified and unfounded. Firstly, the normal distribution, which was heavily depending on the standard deviation, was often assumed in VaR models that couldn't appropriately detect the risk in the tails of the distribution. Secondly, the risk beyond VaR is usually not captured with the VaR model itself. The tail beyond the VaR determines the risk of extreme losses. We suggest to investigate the impact of range based volatility models in the standard VaR model. The empirical analysis is performed on the same dataset as described in section 2.

8.2 VaR model for range based volatility

The VaR model that will utilize the information set included in OHLC data has the form

$$VaR_{\alpha,i,j+1} = \Phi^{-1}(1-\alpha) \cdot \hat{\sigma}_{i,j} \qquad 8\text{-}1$$

This model is without a drift adjustment, as we assume the present value of the expected return to be zero. The function $\Phi^{-1}(\cdot)$ denotes the inverse of the standard normal function with a significance of 1-α. The range based volatility models, $\hat{\sigma}_{i,j}$, that will be compared are presented in section 3.2.1. VaR is calculated for each volatility model *i* and each time step *j*. The empirical analysis of the index dataset is described in chapter 2. The volatility models are calibrated using 10 day history. The results of the VaR assessment are backtested with a set of Coverage tests. The unconditional Coverage (UC) test was introduced by Kupiec (1995) and is considered the first one, which will be complimented with the other two tests. Christoffersen (1998) included the test on the independence of exceedances (IND) to test whether exceedances come in clusters of a first order Markov chain, i.e. a test for consecutive exceedances. Christoffersen (1998) generalized the coverage tests and combined the UC and IND test into one Conditional Coverage (CC*) test. A coverage test is a test of the null hypothesis that the exceedances (π_{obs}) from a particular model are significantly different from the theoretical exceedances ($\pi_{\exp}$).

$$LR_{UC} = \frac{\pi_{\exp}^{n_1}\left(1-\pi_{\exp}\right)^{n_0}}{\pi_{obs}^{n_1}\left(1-\pi_{obs}\right)^{n_0}}, \quad \text{8-2}$$

where $\pi_{\exp} = \alpha$ the expected proportion of exceedances is, $\pi_{obs} = \frac{n}{n_1}$ is the observed proportion of exceedances, n_1 is the observed number of exceedances, $n_0 = n - n_1$ is the number of returns without exceedances and *n* is the sample size of the backtest. The asymptotic distribution of $-2\ln\left(LR_{UC}\right)$ is chi-square with one degree of freedom.

$$LR_{ind} = \frac{\pi_{obs}^{n_1}\left(1-\pi_{obs}\right)^{n_0}}{\pi_{01}^{n_{01}}\left(1-\pi_{01}\right)^{n_{00}}\pi_{11}^{n_{11}}\left(1-\pi_{11}\right)^{n_{10}}} \quad \text{8-3}$$

$$LR_{CC} = \frac{\pi_{\exp}^{n_1}\left(1-\pi_{\exp}\right)^{n_0}}{\pi_{01}^{n_{01}}\left(1-\pi_{01}\right)^{n_{00}}\pi_{11}^{n_{11}}\left(1-\pi_{11}\right)^{n_{10}}}, \quad \text{8-4}$$

where $\pi_{01} = \frac{n_{01}}{n_{00}+n_{01}}$ and $\pi_{11} = \frac{n_{11}}{n_{10}+n_{11}}$. The asymptotic distribution of $-2\ln(LR_{CC})$ is chi-squared with 2 degrees of freedom. The linear relationship between these 3 tests is:

$-2LR_{cc} = -2\ln LR_{uc} - 2\ln LR_{ind}$.

8.3 Empirical results of the VaR application

The results of the VaR assessment for each of the volatility models are shown in Table 8-1 and Table 8-2. The results of the VaR models are divided in four sets. The first set includes the standard deviation, CC, CO and COC. The second set includes the range based volatility models that do not include overnight returns, i.e. HL, Park, RS and GK. The third set includes range based models that incorporate overnight returns, i.e. HL Ext, Park Ext, RS Ext, GK Ext and Yang Zhang. In the fourth set only the results of the TTSE as the unbiased estimator are included. The first three rows of each index show the results of the UC, IND and CC backtests, the fourth row shows the number of overruns and the last row shows the RMSE of the VaR models compared to the return. The RMSE indicates how close the VaR models follow the observed returns.

The first set of VaR models are rejected by the unconditional and conditional coverage tests at both, the 5% and 1%, level for each of the indices. The independence test, however, is not quite rejected. The results of the IND test suggest that there is not enough evidence to detect dependency between the overruns at the 5% and 1% level for all the models and all the considered indices. The total number of overruns varies across the indices. For example, the standard deviation shows, with in total 12 overruns, the least number of overruns for the .WIG index. The .BUX and .CRBX indices show 15 overruns. The CC model shows the least number of overruns for the .WIG index (7) and the CO model shows the least number of overruns for .SOFIX index (8).

In the second set of VaR model, the VaR calculated from Park, RS and GK are rejected by the UC and CC* tests at both, the 5% and 1%, level for each of the indices. Only the HL model from the second set cannot be rejected at neither the 5% nor the 1% level for almost all of the indices. Only for the .PX we observe that both the UC (0.1%) and the CC* (0.1%) tests are rejected at the 1% level. The HL model also shows the least number of overruns and the highest RMSE for all of the indices. This result suggests that the HL model does not follow the returns closely, yet it also ensures that the coverage tests do not reject the hypothesis.

Also in case of the third set of VaR models that incorporate overnight returns we observe that the UC and CC* tests are rejected at the 5% and 1% level. Only HL Ext cannot be rejected at these levels of the coverage tests for all of the indices. Also in this case there is one exception. For .PX we observe that both, the conditional and the unconditional, coverage tests are rejected at the 5% level, while they cannot be rejected at the 1% level. The UC and CC* tests denoted 2.5% and 2.5% respectively. From the other extension models only the Park Ext is not rejected by the UC and CC* tests for the .WIG index at both confidence levels. The UC and CC* tests show 9.7% for both tests. Inclusion of overnight returns has resulted in equal or in a slight decrease of the total number of overruns across all indices.

Table 8-1 Results of unconditional, independence and conditional coverage backtest of the VaR calculations with the range based volatility models.

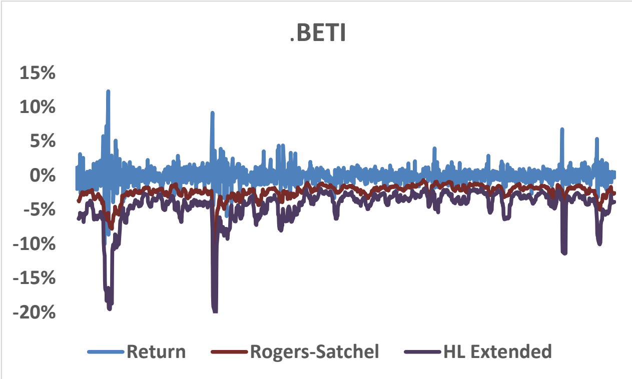
Index	Test	SD	CC	CO	COC	HL	Park	RS	GK	HL*	Park*	RS*	GK*	Yang Zhang	TTSE
.BETI															
	LRuc	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000
	LRind	0.223	0.251	0.311	0.251	1.000	0.044	0.214	0.024	1.000	0.044	0.214	0.024	0.044	0.223
	LRcc	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000
	Overruns	18.000	19.000	21.000	19.000	0.000	22.000	33.000	30.000	0.000	22.000	33.000	30.000	22.000	18.000
	RMSE	0.030	0.031	0.030	0.031	0.045	0.027	0.025	0.025	0.045	0.028	0.026	0.026	0.027	0.029
.BUX															
	LRuc	0.000	0.000	0.000	0.000	0.658	0.000	0.000	0.000	0.658	0.000	0.000	0.000	0.000	0.000
	LRind	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	LRcc	0.000	0.000	0.000	0.000	0.658	0.000	0.000	0.000	0.658	0.000	0.000	0.000	0.000	0.000
	Overruns	15.000	15.000	22.000	10.000	1.000	17.000	18.000	18.000	1.000	12.000	14.000	13.000	9.000	21.000
	RMSE	0.039	0.039	0.035	0.039	0.055	0.033	0.032	0.032	0.058	0.037	0.037	0.037	0.037	0.030
.CRBX															
	LRuc	0.000	0.000	0.000	0.000	0.289	0.000	0.000	0.000	0.289	0.000	0.000	0.000	0.000	0.000
	LRind	1.000	1.000	1.000	1.000	1.000	1.000	0.195	1.000	1.000	1.000	0.195	1.000	1.000	0.107
	LRcc	0.000	0.000	0.000	0.000	0.289	0.000	0.000	0.000	0.289	0.000	0.000	0.000	0.000	0.000
	Overruns	15.000	15.000	15.000	15.000	3.000	11.000	17.000	15.000	3.000	11.000	17.000	14.000	13.000	13.000
	RMSE	0.019	0.019	0.019	0.020	0.029	0.018	0.017	0.017	0.030	0.019	0.018	0.018	0.019	0.018

Table 8-2 Results of unconditional, independence and conditional coverage backtest of the VaR calculations with the range based volatility models.

Index	Test	SD	CC	CO	COC	HL	Park	RS	GK	HL*	Park*	RS*	GK*	Yang Zhang	TTSE
.PX															
	LRuc	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.025	0.000	0.000	0.000	0.000	0.000
	LRind	0.857	0.565	0.452	0.132	1.000	0.850	0.407	0.771	1.000	1.000	1.000	1.000	1.000	0.691
	LRcc	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.025	0.000	0.000	0.000	0.000	0.000
	Overruns	35.000	28.000	41.000	29.000	7.000	42.000	52.000	44.000	5.000	27.000	34.000	31.000	32.000	70.000
	RMSE	0.024	0.024	0.021	0.024	0.032	0.019	0.018	0.018	0.034	0.022	0.021	0.021	0.022	0.017
.SOFIX															
	LRuc	0.000	0.000	0.000	0.001	0.674	0.000	0.000	0.000	0.674	0.001	0.000	0.000	0.005	0.088
	LRind	0.317	0.317	0.037	0.027	1.000	0.001	0.001	0.001	1.000	0.027	0.025	0.255	0.019	0.008
	LRcc	0.000	0.000	0.000	0.000	0.674	0.000	0.000	0.000	0.674	0.000	0.000	0.000	0.000	0.002
	Overruns	21.000	21.000	8.000	7.000	1.000	8.000	9.000	9.000	1.000	7.000	19.000	19.000	6.000	4.000
	RMSE	0.024	0.025	0.024	0.025	0.038	0.023	0.023	0.023	0.039	0.024	0.024	0.024	0.025	0.027
.WIG															
	LRuc	0.000	0.001	0.000	0.006	1.000	0.000	0.000	0.000	1.000	0.097	0.001	0.006	0.000	0.000
	LRind	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000	1.000
	LRcc	0.000	0.001	0.000	0.006	1.000	0.000	0.000	0.000	1.000	0.097	0.001	0.006	0.000	0.000
	Overruns	12.000	7.000	15.000	6.000	0.000	15.000	18.000	16.000	0.000	4.000	7.000	6.000	12.000	11.000
	RMSE	0.034	0.034	0.030	0.035	0.047	0.028	0.028	0.028	0.051	0.034	0.033	0.033	0.033	0.029

Interestingly, the VaR calculated from the unbiased volatility estimator, the TTSE, is rejected by the UC and CC* tests at both, the 5% and 1%, level for almost of the indices. Only for .SOFIX we cannot reject the UC test (8.8%) at the 5% level. For most of the indices there is not sufficient evidence to reject the IND test at the 5% or 1% level. For .WIG the independence test is rejected at both levels, while the independence and conditional coverage tests are rejected for the .SOFIX index at the 5%, but not quite at the 1% level. The RMSE denotes that for most of the indices the VaR based on the TTSE model follows the observed returns closely.

The obtained results show that the VaR models based on the HL or HL Ext cannot be rejected in almost all of the presented cases. In fact the VaR model based on HL is only rejected in case of the .PX index, while for the rest of the indices the results between the HL and HL Ext model are almost the identical. Both VaR models also resulted in the least number of overruns throughout the empirical analysis. Unsurprisingly the HL Ext model also has the highest RMSE across all of the indices. The RMSE on the other hand measures the overall distance between the VaR estimates and the observed returns. VaR models that cannot be rejected at the 5% nor at the 1% confidence level and are at the same time 'far away' (indicated with a relative high RMSE) from the observed returns, can be considered overestimated. A good VaR model must first of all serve its purpose. If its purpose is never to be overrun then the RMSE is of less importance. However, if its purpose is to minimize the possibility of an overrun and at the same time not to be too far away from the observed returns, the RMSE becomes an important measure to consider.



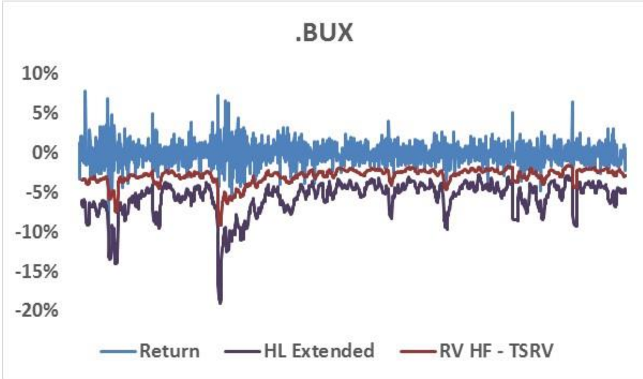


Figure 8-1 shows the history of different VaR models against the observed returns.

It has been noted in the Roger and Satchell and the Garman and Klass range-based volatility models have an overall low RMSE (.BETI and .WIG), GK (.CRBX and .SOFIX) and TTSE (.BUX and .PX) VaR models. The RMSE of the HL and HL Ext, on the other hand, is much higher indicating that the HL VaR models are not following the realized returns close enough.

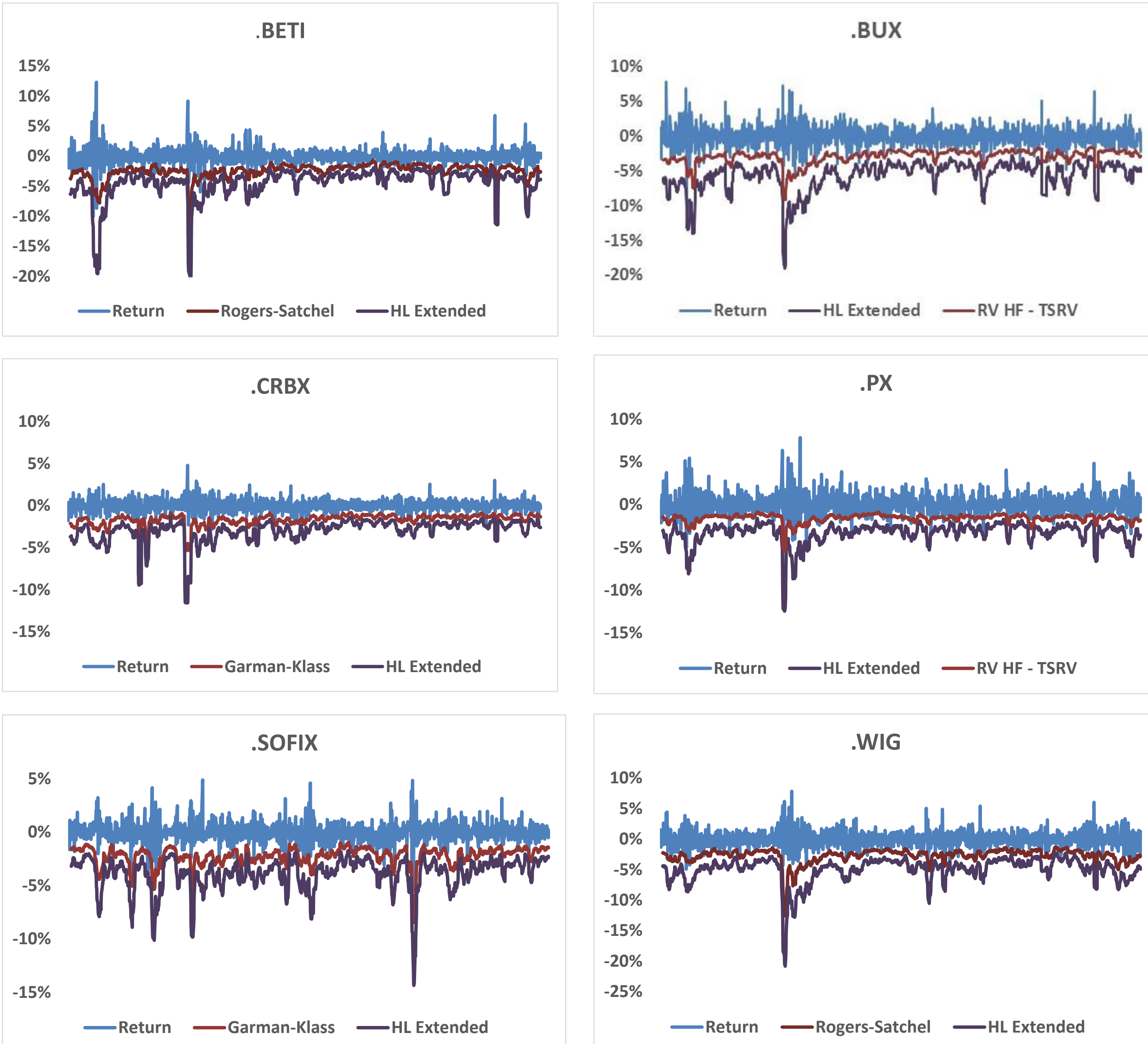


Figure 8-1 VaR models with the lowest (red) and highest (purple) RMSE versus the index returns.

9 APPLICATION II - PORTFOLIO OPTIMIZATION

Part of this chapter has been published in the International Journal of Mathematical and Computational Sciences, Poklepović, Aljinović, Matkovic (2015)

The second application concerns a portfolio optimization exercise based on individual stock prices of the .CRBX index. The Parkinson range-based volatility model is utilized to build an optimal portfolio consisting of a limited number of stock from the .CRBX index.

9.1 Portfolio optimization and Markowitz modern portfolio theory

Modern Portfolio Theory was introduced by Nobel Laureate Harry Markowitz (1952) in his seminal paper which changed the way portfolios were managed until then. This theory focuses on portfolio diversification and risk control. Investors form portfolios according to the mean variance efficiency criteria. This means that investors maximize their return across all possible portfolios and accept the risk according to their risk aversion. Markowitz describes a risk averse investor as a subject who prefers a higher return versus a lower return and who at the same time is prepared to accept more risk if such investment increases the expected return. Such an investor optimizes the expected portfolio return given the portfolio risk. Modern portfolio theory is based on the efficient frontier (EF) of investments, i.e. the spine of portfolios with maximum expected return across all possible portfolios given a certain amount of portfolio risk. The portfolio risk is of crucial information to the investor and therefore needs to be quantified. The volatility of the portfolio return is often considered as the risk of concern. Since volatility is not observable it needs to be estimated. Markowitz proposed to quantify portfolio risk by means of the volatility of financial assets. He used the standard deviation of financial assets as a simple measure of risk and the lower semi-variance as the more complex estimator. The lower semi-variance is according to Markowitz the only volatility estimator in which a rational investor might be interested in. On the other side the standard deviation has become one of the most popular risk estimators in practice due to the simplicity in using and understanding this measure in portfolio management. However, both estimators use only one single daily observable price change to determine the volatility of financial returns. An alternative volatility estimator used in this paper is based on high frequency data. Volatility estimated by means of high frequency data is also called realized volatility and can be considered unbiased. In practice, however, the implementation of high frequency data is limited by several reasons. First of all, high frequency data is not available for all securities. This is especially true for securities traded in emerging markets where the trading volume is often insufficient as the data frequency becomes smaller. Secondly, as the frequency becomes smaller microstructure effects emerge which induce an upward bias in the estimated volatility. Thirdly, there is a serious calculation complexity due to the extensive amount of data that is required for estimating the daily volatility or the variance-covariance matrix. For example to calculate the volatility of 250 trading days based on 5-minute interval observations around 24.000 intraday price observations are required. Moreover for estimating the EF of a portfolio consisting of 20 assets more than a million observations will be required. A more practical methodology to estimate the intraday volatility is by means of open, high, low and closing prices (OHLC). This paper uses the Parkinson (1980) range-based volatility estimator for extreme price jumps, which are characteristic for emerging markets like Croatia. A significant shortcoming however, of the range-based volatility estimator is that no multivariate analogue of the intraday range exists, which means that the estimation of the variance-covariance matrix is not straightforward. A simple estimator of the conditional variance-covariance matrix of returns that was proposed by Harris and Yilmaz (2007). This methodology is used to construct the EF based on the range-based volatility estimator. This paper compares EF based on 3 different volatility estimators using a portfolio of stocks from the Croatian Stock Market.

The outline of the remainder of the paper is as follows. Section 2 reviews the literature on modern portfolio theory with focus on the literature on different approaches to estimating the volatility. Section 3 describes the modern portfolio theory, which is the basis of this research. Section 4 presents the lower semi-variance approach in estimating the efficient frontier and section 5 the intraday volatility approach. The stock price data is described in section 6. The results of the empirical research are presented in section 7. Section 8 concludes.

In his seminal paper, Markowitz (1952) describes Modern Portfolio Theory in a quantitative model that solves the complex problem of capital allocation across assets in such a way that it minimizes the variance of the portfolio given an expected return. Markowitz proposed a simple square root optimisation that results in the mean-variance efficient portfolio and suggested to use the standard deviation for estimating the portfolio volatility. The standard deviation is a statistically correct estimator of the volatility of returns if the observed time series are derived from a normal distribution. This, however, is not always the case. One of the stylized facts of financial returns as described in Cont (2001) is the non-normality of financial returns. The distribution often 'suffers' from positive skewness and leptokurtosis. Other stylized facts include amongst others heteroscedasticity and time varying correlations of financial returns. Therefore the standard deviation, which assumes normality by default, is expected to underestimate the true volatility of the distribution. Motivated by the definition of risk, as a financial loss or downside risk, Markowitz (1952) proposes a new definition of risk considering only the negative results.

The lower semi-variance measures the dispersion of the returns below a given target return. Markowitz explains that the usage of the lower semi-variance is justified by two reasons. Firstly, rational investors are only interested in limiting the volatility that can cause a negative result. Secondly, if the financial time series are not normally distributed then the standard deviation will underestimate the true risk of the portfolio. In these cases the lower semi-variance, as a measure of downside risk, should be used instead. It is shown in James (1970) that there is a great support in the market for using the lower semi-variance as a risk measure. Investors are more sensitive to losses below a certain threshold then to gains beyond a certain threshold. In¸ Bawa and Vijay (1975) the formula for lower semi-variance is generalized and defined as the lower partial moment (LPM). Four different LPM volatility estimators are compared in Konno, Waki and Yuuki (2002) and it is shown that the LPM proposed by Markowitz is suitable for controlling risk when the distribution of the assets is not normal.

Both estimators that were proposed by Markowitz use only one single daily price observation to determine the variance-covariance matrix. This means that all other price observations that are available when high frequency data a used are ignored.

One of the recent theories focusing on volatility estimators are described in the literature of high frequency data. The realized volatility estimator is proposed in Dacarogna, Muller, Olsen and Pictet (1998), which is the squared sum of intraday returns. According to Andersen, Bollerslev, Diebold and Labys (2001) this volatility estimator is theoretically unbiased when the frequency sample goes to zero, but will in turn induce microstructure effects. The realized volatility is estimated by using all market available intraday information. Another practical disadvantage of this method is that it requires an extensive amount of intraday price observations for estimating an EF. A reasonable alternative to using high frequency data is to use volatility estimators that require only 4 standard available intraday price observations, i.e. the OHLC estimators. OHLC estimators, generally, assume that asset prices follow a Geometric Brownian Motion (GBM) i.e. the price of the asset on day t is independent of the price of the same asset on day t-1 and that the price of the assets are stochastic through time. GBM without drift is assumed in Parkinson (1980) and it proposes a range based volatility estimator. This estimator uses the maximum difference between the maximum and the minimum intraday price for estimating the volatility. The open and closing prices are included in Garman and Klass (1980) and they propose an estimator, which uses all four OHLC intraday price observations. An estimator that follows a GBM with drift is proposed in Rogers and Satchel (1991). This estimator is useful when the drift is non zero. Significant differences between OHLC estimators that are popular in the literature are found in Duque and Paxson (1997) and they conclude that the choice of the OHLC volatility estimator is important. OHLC volatility estimators that are popular in the literature are compared in Arnerić, Matković and Čorić (2018), against the unbiased high frequency based volatility estimator. They show that the Parkinson range-based volatility estimator is the least biased estimator for estimating the volatility of the Croatian Stock market compared to other OHLC volatility estimators. The comparison is performed against the high frequency based realized volatility which is the theoretically unbiased volatility estimator. The data used in their research spans a period of 5 years and includes the recent credit and bank crisis of 2007 and 2008. They confirm the findings of Duque and Paxson (1997) by means of loss functions and time varying conditional correlations and conclude that the OHLC volatility estimators are significantly different from each other. This paper follows the results of Arnerić, Matković and Čorić (2018) and uses the Parkinson range-based volatility estimator in estimating the intraday volatility. The conditional variance-covariance matrix proposed in Harris and Yilmaz (2007) is used to construct the EF by means of mean-variance.

EF are compared in Foo and Eng (2009), Sing and Ong (2000) and Sivitanides (1998) based on the standard deviation and the lower semi-variance and conclude that it is possible to construct an EF based on the lower semi-variance that lies on the left side of the EF based on the simple standard deviation. They conclude that this EF is stochastically dominant compared to the standard deviation proposed by Markowitz. It is possible to reduce the risk of a portfolio by using the lower semi-variance as a measure of the portfolio volatility (Foo and Eng (2009)). According to Cheng and Woverton (2001) it is not possible to compare EF based on different volatility estimators since the risk estimators are not identical, i.e. the x-axis on the mean-variance coordinate system is different. They conclude that the only meaningful way of comparing EF is by ex-post analysis and that the location of the EF on the mean-variance coordinate system does not add valuable information.

This research compares EF based on different volatility estimators: standard deviation, lower semi-variance and the intraday volatility estimator. The questions of interest are whether the volatility estimator influences the location of the efficient frontier on the mean-variance coordinate system and whether the location of the efficient frontier on the mean-variance coordinate system determines the performance of the efficient portfolios by the ex-post analysis.

According to Modern Portfolio Theory (MPT) investors use mean-variance optimization to construct an efficient portfolio. MPT relies on the following assumptions: the investment horizon is one period (one month, one year, etc.); investors optimize their expected return across all possible portfolios; the expected portfolio return depends on the expected return and the risk of the investment; investors are rational and prefer a higher return compared to a lower return, and also have aversion to risk; there is no tax, no inflation and there is no transaction or other costs involved; all investors have free and unlimited access to relevant information at the same time; all stocks are infinitely divisible. This set of assumptions creates a theoretical world in which investors operate according to MPT. This world is different from the real world, but incorporates almost all elements average investors take into account when making investment decisions. According to MPT investors will spread their portfolio to divers or control the risk and at the same time they want to maximize their expected return. Optimization is based on mean-variance efficiency, which means that, given a predetermined portfolio risk, investors will choose the portfolio that maximizes their return. The standard deviation is a popular volatility estimator that requires only one price observation per day. This estimator is symmetrical and assumes that the returns follow a normal or multivariate normal distribution. Considering that every efficient portfolio has the highest revenue along with defined rate of risk c, mathematically we may define efficient portfolio as follows (Aljinović, Marasović and Šego (2011)):

$$\max\{E(\mathrm{R}_{\pi})\}.$$

Subject to:

$$\sigma_{\pi} \leq c \qquad 9\text{-}1$$

and

$$\sum_{i=1}^{n} \pi_i = 1, \pi_i \geq i \in \{1,2,...n\}$$

The expected portfolio return is defined as

$$E(R_{\mu}) = \sum_{i=1}^{n} \pi_i E(R_i) = \pi' \cdot E(R) = E(R)' \cdot \pi \qquad 9\text{-}2$$

and the portfolio risk as

$$\sigma_{\pi} = \sqrt{\pi' E(R_i)} = \pi' \cdot E(R) = E(R)' \cdot \pi \qquad 9\text{-}3$$

$$\sigma_{\pi} = \sqrt{\pi' \cdot S \cdot \pi} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \pi_i \pi_j \sigma_{ij}} \qquad 9\text{-}4$$

$E(R)$ denotes the vector column of expected returns, $E(R_i)$ the expected return of the stock i, π the vector column of weights of a stock in a portfolio, π_i the weight of a stock i in portfolio π, S is the variance-covariance matrix, $\sigma_{i,j}$ the covariance of returns of stocks i and j, $E(R_{\pi})$ expected portfolio return, σ_{π} standard deviation of the portfolio and n the number of stocks.

9.2 Range based volatility approach

The standard deviation and the lower semi-variance use one single daily price observation in determining the volatility of the portfolio. All other relevant information available to the investor is ignored. Intraday volatility estimators use more daily price observations in computing the volatility. These estimators do not rely on the normal distribution. It is shown in Dacarogna, Muller, Olsen and Pictet (1998) that the unbiased volatility estimator could be constructed by means of high frequency data. The trading volume in emerging markets is often insufficient to ensure high frequency data for all required stocks. Due to limitations, we follow the work of Arnerić, Matković and Šorič (2018) who showed that the Parkinson range-based volatility estimator is the least biased OHLC estimator when estimating the volatility of the Croatian Stock Market based on the Loss Function approach as showed in section 7.3. The Parkinson range based volatility estimator uses two intraday price observations to determine the spread: the highest and the lowest intraday observations.

The range-based volatility estimator is given by:

$$\sigma_{ii,t}^{Range} = \sqrt{\frac{1}{4\ln 2} \cdot \ln\left(\frac{H_i}{L_i}\right)^2} \tag{9-5}$$

In equation (8) H denotes the highest and L denotes the lowest observed intraday price. Using the highest and the lowest price observations Parkinson proposes a volatility estimator for high volatile markets. This estimator follows a GBM without drift and uses only extreme price movements to calculate the volatility. The portfolio risk as defined in equation (5) requires the variance-covariance matrix for input. The off-diagonal elements of the variance-covariance matrix of the range-based volatility estimator are not directly observable. A simple model that is based on the exponentially weighted moving average (EWMA) to estimate the off-diagonal elements of the variance-covariance matrix when using the range-based volatility estimator is proposed in Harris and Yilmaz (2007). This model combines the range-based and the return-based approaches. The return-based volatility estimator is given by:

$$\sigma_{ii,t}^{R} = \sqrt{\ln\left(\frac{p_{i,t}}{p_{i,t-1}}\right)^2}$$

where $p_{i,t}$ is the price of stock i on day t.

The estimator is based on the multivariate EWMA model of the conditional variance-covariance matrix given by

$$\hat{\sigma}_{ij,t}^{R} = \lambda \hat{\sigma}_{ij,t-1}^{R} + (1-\lambda)\sigma_{ij,t-1}^{R}; i, j = 1,...,n \qquad 9\text{-}6$$

where λ is the single decay factor, which is typically set to 0.94, estimated by JP Morgan as the average value of decay factor that minimizes the mean square error of daily out-of-sample conditional volatility forecasts for a wide range of assets.

The diagonal and off-diagonal elements of the range-based estimator of the conditional variance-covariance matrix are given by

$$\hat{\sigma}_{ij,t}^{Range} = \begin{cases} \lambda \hat{\sigma}_{ii,t-1}^{Range} + (1-\lambda)\sigma_{ii,t-1}^{Range}; i = j \\ \rho_{ij}^{R}\sqrt{\hat{\sigma}_{ii,t}^{Range}\hat{\sigma}_{jj,t}^{Range}}; i \neq j \\ i, j = 1,...,n \end{cases} \qquad 9\text{-}7$$

where

$$\rho_{ij}^{R} = \frac{\sigma_{ij,t}^{R}}{\sqrt{\hat{\sigma}_{ii,t}^{R}\hat{\sigma}_{jj,t}^{R}}}; i, j = 1,...,n \qquad 9\text{-}8$$

Finally, the elements of the variance-covariance matrix of the range-based volatility model are calculated by

$$\sigma_{ij} = \hat{\sigma}_{ii,t}^{Range}\hat{\sigma}_{jj,t}^{Range}\rho_{ij}^{R} \qquad 9\text{-}9$$

Now, when the range-based variance-covariance matrix is known, we proceed with steps (1) to (5) to calculate the mean variance portfolio.

9.3 Data description

The portfolios constructed in this research consist of an investment in 10 stocks from the CRBX index. The data spans from 12th March 2013 to 13th December 2013 and counts 191 price observations. The following stocks are included: AD Plastik d.d. (ADPL), Atlantska Plovidba d.d. (ATPL), Belje d.d. (BLJE), Djuro Djaković Holding d.d. (DDJH), Dalekovod d.d. (DLKV), Valamar Adria Holding d.d. (DOMF), Ericsson Nikola Tesla d.d. (ERNT), Hrvatski Telekom d.d. (HT), Ingra d.d. (INGR) and Vupik d.d. (VPIK).

Table 9-1 shows the descriptive statistics including the sample size, minimum, maximum, expected returns, the volatility at the end of the investment period for each volatility estimator, skewness, kurtosis and the Jarque-Bera test for normality of returns. The descriptive statistics shows that all assets show asymmetric behaviour and leptokurtosis, i.e. deviation from the normal distribution. The Jarque-Bera test shows that none of the stocks follows a normal distribution.

Table 9-1 shows the descriptive statistics for a selection of stocks

	ADPL	ATPL	BLJE	DDJH	DLKV
N	190	190	190	190	190
Min	-0.02840	-0.06480	-0.05200	-0.05730	-0.30800
Max	0.04120	0.07310	0.10100	0.08250	0.26000
Expected return	0.00000	0.00133	-0.00203	-0.00145	-0.00233
Variance	0.00007	0.00077	0.00037	0.00053	0.00328
Standard deviation	0.00864	0.02770	0.01920	0.02290	0.05730
Lower semi-variance	0.00004	0.00033	0.00016	0.00022	0.00162
Lower semi-SD	0.00604	0.01810	0.01250	0.01480	0.04030
Range-based volatility	0.00477	0.02110	0.02040	0.02720	0.05490
Skewness	0.24	0.49	1.04	0.69	-0.18
Kurtosis	3.66	0.16	5.04	1.37	7.20
Jarque-Bera	5.18	71.51	66.93	36.02	140.47

	DOMF	ERNT	HT	INGR	VPIK
N	190	190	190	190	190
Min	-0.03410	-0.12800	-0.11300	-0.07500	-0.07350
Max	0.02890	0.04220	0.03290	0.14400	0.07240
Expected return	-0.00031	-0.00025	-0.00111	-0.00154	-0.00152
Variance	0.00013	0.00021	0.00014	0.00094	0.00040
Standard deviation	0.01140	0.01450	0.01180	0.03070	0.02000
Lower semi-variance	0.00007	0.00014	0.00010	0.00036	0.00019
Lower semi-SD	0.00823	0.01180	0.00998	0.01910	0.01390
Range-based volatility	0.00947	0.00675	0.00619	0.03020	0.01740
Skewness	-0.19	-3.44	-4.51	1.21	0.10
Kurtosis	0.39	31.17	43.18	4.37	1.65
Jarque-Bera	55.01	6,659.13	13,422.71	61.05	14.83

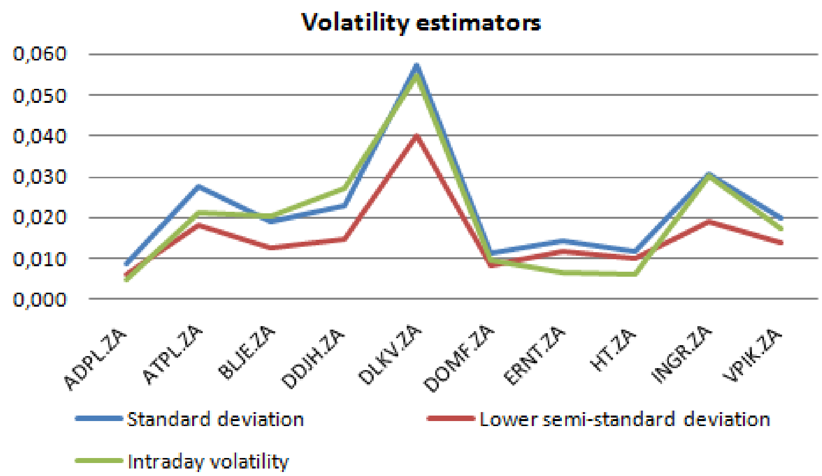


Figure 9-1 Estimated volatility over time using different volatility estimators.

Figure 9-1 shows values of the three observed volatility estimators for each stock. It can be concluded that, on average, the highest volatility is estimated by standard deviation and the intraday range-based volatility estimator and that the lowest volatility is estimated by the semi-standard deviation.

9.4 Empirical results of the Portfolio Optimization application

The locations of the EF on the mean-variance coordinate system and the performance of the three different volatility estimators in the ex-post, or out-of-sample analysis, are compared.

In the first part of the analysis, the EF is computed at 20 different risk levels for the portfolios using mean-variance, lower semi-variance and intraday range-based volatility approach. The appropriate weights, returns and standard deviations are presented in Tables II, III and IV.

Table 9-2 Efficient Portfolios Using Mean-Variance Model

ADPL	ATPL	BLJE	DDJH	DLKV	DOMF	ERNT	HT	INGR	VPIK	SD (%)	Return (%)
0.345	0.059	0.035	0.040	0.002	0.187	0.121	0.177	0.000	0.035	0.568	-0.039
0.582	0.168	0.000	0.000	0.000	0.136	0.114	0.000	0.000	0.000	0.700	0.015
0.634	0.228	0.000	0.000	0.000	0.052	0.086	0.000	0.000	0.000	0.800	0.027
0.662	0.277	0.000	0.000	0.000	0.000	0.061	0.000	0.000	0.000	0.900	0.035
0.655	0.326	0.000	0.000	0.000	0.000	0.020	0.000	0.000	0.000	1.000	0.043
0.628	0.372	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.100	0.050
0.582	0.418	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.200	0.056
0.539	0.461	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.300	0.061
0.498	0.502	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.400	0.067
0.458	0.542	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.500	0.072
0.420	0.580	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.600	0.077
0.382	0.618	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.700	0.082
0.345	0.656	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.800	0.087
0.308	0.692	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.900	0.092
0.271	0.729	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	0.097
0.235	0.765	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.100	0.102
0.200	0.801	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.200	0.107
0.164	0.836	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.300	0.111
0.094	0.907	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.500	0.121
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.768	0.133

The results of the three different volatility estimators show very similar range for the return of portfolios. However, the highest risk is found with mean-variance model. It ranges from 0.57%, when diversifying portfolio and investing in all but one stock (INGR is not included in portfolio) yielding a return of -0.04%, to 2.77% when investing in only one share (ATPL), yielding a return of 0.13%. When considering lower semi-variance, the risk ranges from 0.45%, when diversifying portfolio and investing in ADPL, ATPL, DDJH, DOMF, ERNT and HT yielding 0.00% return, to 1.81% when investing in only one stock (ADPL) yielding return of 0.13%.

Table 9-3 Efficient Portfolios Using Lower Semi-Variance Model.

ADPL	ATPL	BLJE	DDJH	DLKV	DOMF	ERNT	HT	INGR	VPIK	SD (%)	Return (%)
0.527	0.122	0.000	0.003	0.000	0.201	0.080	0.068	0.000	0.000	0.450	0.000
0.527	0.122	0.000	0.003	0.000	0.201	0.080	0.067	0.000	0.000	0.450	0.000
0.531	0.124	0.000	0.001	0.000	0.201	0.080	0.064	0.000	0.000	0.452	0.001
0.571	0.148	0.000	0.000	0.000	0.190	0.072	0.018	0.000	0.000	0.472	0.010
0.602	0.195	0.000	0.000	0.000	0.142	0.061	0.000	0.000	0.000	0.507	0.020
0.627	0.251	0.000	0.000	0.000	0.072	0.050	0.000	0.000	0.000	0.564	0.030
0.654	0.307	0.000	0.000	0.000	0.000	0.039	0.000	0.000	0.000	0.637	0.040
0.636	0.342	0.000	0.000	0.000	0.000	0.022	0.000	0.000	0.000	0.681	0.045
0.618	0.376	0.000	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.728	0.050
0.550	0.450	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.835	0.060
0.475	0.525	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.953	0.070
0.400	0.600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.079	0.080
0.325	0.675	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.210	0.090
0.287	0.713	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.276	0.095
0.250	0.750	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.344	0.100
0.175	0.825	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.482	0.110
0.100	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.621	0.120
0.025	0.975	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.763	0.130
0.002	0.998	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.806	0.133
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.809	0.133

Table 9-4 Efficient Portfolios Using Intraday Volatility Model.

ADPL	ATPL	BLJE	DDJH	DLKV	DOMF	ERNT	HT	INGR	VPIK	SD (%)	Return (%)
0.177	0.062	0.044	0.038	0.024	0.196	0.155	0.200	0.041	0.063	0.309	-0.060
0.199	0.073	0.030	0.022	0.017	0.198	0.172	0.201	0.036	0.052	0.310	-0.050
0.221	0.084	0.016	0.006	0.010	0.200	0.190	0.203	0.030	0.041	0.315	-0.040
0.245	0.094	0.002	0.000	0.003	0.202	0.208	0.199	0.022	0.026	0.324	-0.029
0.274	0.108	0.000	0.000	0.000	0.203	0.223	0.181	0.010	0.001	0.338	-0.019
0.300	0.134	0.000	0.000	0.000	0.196	0.235	0.134	0.001	0.000	0.361	-0.009
0.324	0.164	0.000	0.000	0.000	0.188	0.245	0.079	0.000	0.000	0.394	0.001
0.347	0.194	0.000	0.000	0.000	0.181	0.255	0.023	0.000	0.000	0.436	0.011
0.343	0.246	0.000	0.000	0.000	0.165	0.247	0.000	0.000	0.000	0.485	0.021
0.319	0.313	0.000	0.000	0.000	0.143	0.226	0.000	0.000	0.000	0.548	0.032
0.294	0.380	0.000	0.000	0.000	0.121	0.205	0.000	0.000	0.000	0.622	0.042
0.270	0.447	0.000	0.000	0.000	0.099	0.184	0.000	0.000	0.000	0.703	0.052
0.246	0.514	0.000	0.000	0.000	0.077	0.162	0.000	0.000	0.000	0.789	0.062
0.222	0.582	0.000	0.000	0.000	0.055	0.141	0.000	0.000	0.000	0.878	0.072
0.198	0.649	0.000	0.000	0.000	0.033	0.120	0.000	0.000	0.000	0.970	0.082
0.174	0.716	0.000	0.000	0.000	0.011	0.099	0.000	0.000	0.000	1.063	0.093
0.144	0.785	0.000	0.000	0.000	0.000	0.072	0.000	0.000	0.000	1.159	0.103
0.107	0.855	0.000	0.000	0.000	0.000	0.038	0.000	0.000	0.000	1.255	0.113
0.071	0.925	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000	1.354	0.123
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.453	0.133

The lowest risk is measured with intraday range-based estimator ranging from 0.31%, when diversifying risk and investing in all the stocks with different weights yielding the negative return of -0.06%, to 1.45% when investing in ATPL yielding the return of 0.13%. It can be concluded that perhaps intraday range-based volatility estimator underestimates the risk compared to the two other volatility estimators. However, the results of the ex-post analysis test the performances of the models.

The computed efficient frontiers are plotted on the mean-variance coordinate system for the mean-variance model, lower semi-variance and intraday range-based volatility model and are presented in Figure 9-2, Figure 9-3 and Figure 9-4.

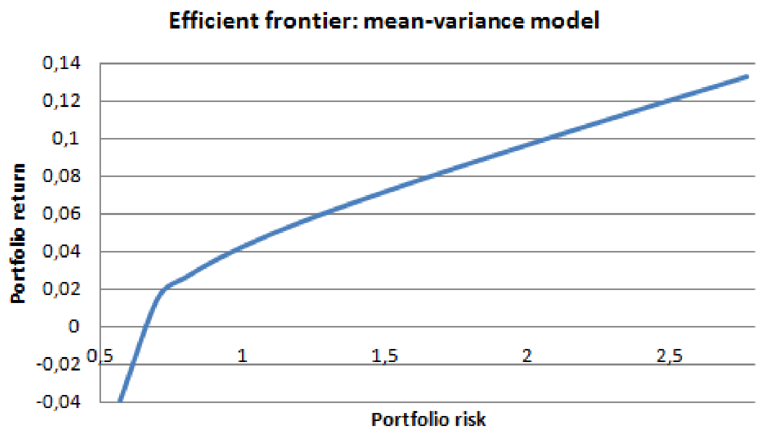


Figure 9-2 the efficient frontier based on the mean-variance model.

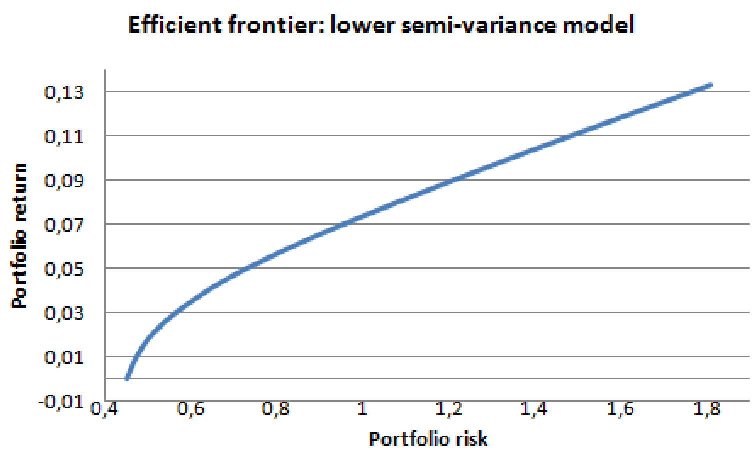


Figure 9-3 shows the efficient frontier based on the lower semi-variance model.

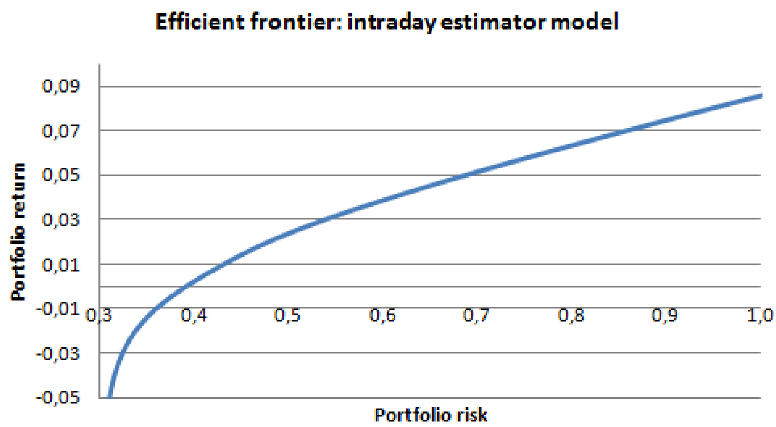


Figure 9-4 shows the efficient frontier based on the intraday estimator model.

In the second step, an ex-post analysis is performed by investing in a portfolio of stocks using the calculated portfolio weights. The stock returns on the next trading day and the calculated weights are used to calculate for each model the portfolio returns. The results are presented in Table 9-5.

Table 9-5 Expected portfolio return in percentages for the three different volatility estimators.

	mean-variance	Lower semi-variance	Range-based volatility model
1	0.5039	0.3518	0.8597
2	0.0791	0.3513	0.7389
3	-0.2381	0.3466	0.6181
4	-0.4154	0.3030	0.5066
5	-0.3587	0.1332	0.4460
6	-0.3007	-0.1211	0.3999
7	-0.2405	-0.3796	0.3759
8	-0.1840	-0.3372	0.3537
9	-0.1301	-0.2947	0.3349
10	-0.0779	-0.1983	0.3185
11	-0.0270	-0.0996	0.3021
12	0.0228	-0.0009	0.2857
13	0.0718	0.0978	0.2693
14	0.1202	0.1471	0.2529
15	0.1681	0.1965	0.2365
16	0.2155	0.2952	0.2201
17	0.2625	0.3939	0.2536
18	0.3092	0.4926	0.3400
19	0.4019	0.5222	0.4264
20	0.5249	0.5242	0.5249

Assuming that an equal amount of money is invested by 20 investors with different risk aversions that result in the 20 portfolios as presented in tables 9-2, 9-3 and 9-4 given the defined risk measures, the highest return would be obtained if the intraday volatility estimator approach was used. This would investment strategy would yield in a positive return for all risk aversions, while for portfolios 1 to 15 it would yield in the highest portfolio return amongst all risk measures ranging from 0.88% when diversifying risk to 0.52% when investing in only one stock, i.e. ADPL. When the investment is based on the mean-variance or lower semi-variance approach, it yields both positive and negative returns, depending on the risk aversion. Moreover, it yields lower positive returns for more diversified portfolios than the intraday volatility approach. Notice that the last portfolio considers a 100% investment in a single stock, i.e. ADPL, and thereby denotes the portfolio with the highest risk. This portfolio will earn the same amount regardless of the chosen volatility estimators.

Given the mean-variance coordinates of the EF and the performance of the 1-day ex-post analysis we conclude that the range-based volatility model outperforms both the mean-variance and the lower semi-variance models when constructing EF.

According to Markowitz, rational investors are only interested in the lower semi-variance because this estimator measures the risk of losses below a certain threshold, i.e. losses of interest to the investor. Rational investors are concerned about losses, because they want to control their portfolio risk at every point in time. When financial returns do not follow a normal distribution the standard deviation can be replaced by the lower semi-variance. Since both estimators use a single daily price observation in estimating the volatility, intraday volatility estimators can be considered as an alternative. According to Dacarogna intraday volatility estimators are assumed to be unbiased. However, high frequency data induce microstructure effects and also some practical limitation since they do not exist for all assets. The range based intraday volatility estimator has gained interest in recent literature. It is a more efficient estimator than the daily squared close-to-close return and it is relatively robust to microstructure effects. However, since there is no multivariate analogue of the range-based volatility estimator the conditional variance-covariance matrix is estimated by a EWMA-based model, which forms the basis for the mean-variance portfolio estimation. The EF are constructed based on all three volatility estimators. Their performances are compared in the next out-of-sample trading day.

The EF based on these three types of volatility measures show different levels of expected returns, portfolio risk and portfolio diversification. Thus, the EF differs in location on the mean-variance coordinates. The results of the three different volatility estimators show very similar range for the return of portfolios. However, the highest risk is found with mean-variance model and the lowest risk is measured with intraday estimator.

The efficient portfolios based on the intraday range-based volatility estimator outperforms the alternative volatility estimators for most risk levels when considering the investment in these portfolios and the returns on the next trading day.

The results of the portfolio estimation show that the choice of the risk estimator is important in constructing the EF since the portfolio weights differ and thus the choice of the investment.

For further research, we suggest to extend this theoretical research by including a longer period and to include more volatile periods like the recent credit crisis of 2007 and 2008. Intraday volatility has the interesting property of using multiple intraday observations to determine the daily volatility. According to Markowitz rational investors are only interested in the risk of a negative return. Therefore it would be interesting to investigate the performance of a semi-variance version of the range-based volatility model on a set of financial assets.

10 CONCLUSION

The scientific focus in improving models for estimating the volatility of financial markets has been increasing over the past decades and has resulted in an expansion and further enrichment of the already broad literature in this field. To a large extend this increased scientific interest can be prescribed to the increasing availability of high frequency price observations in the past few decades. This has shed some new light to already existing theory of Realized Volatility, which has engraved a new direction in estimating the volatility of financial markets. This theory has opened the windows for a vast increase in scientific research papers, which have focused on estimating the Integrated Volatility. Some of these research papers propose more efficient and unbiased Integrated Volatility estimators. More or less all of the research papers that used empirical data focused the research on the main global markets. For as far as the Author is aware the focus on European Emerging Stock Markets in the context of estimating daily Realized Volatility has been relatively poor in providing a broad overview across several East European Stock markets.

This dissertation distinguishes itself from the existing literature by making a contribution to already existing volatility models. The Realized Volatility as well as the unbiased estimator of the Integrated Volatility, the Two Times Scale Estimator, are calculated for a vast number of European emerging stock markets. The investigation of this Thesis also considers a wide range of parsimonious range-based volatility models that utilize only a limited number of intraday price observations. Some of these models have been extended for overnight returns, since the overnight price changes (also referred to as overnight jumps) may have a significant impact on the preciseness of the estimator. The empirical research is based on 6 East European Stock Markets (.BETI, .BUX, .CRBX, .PX, .SOFIX and .WIG) and provides a more complete overview of the performance of various volatility models on these different markets.

This research has investigated a vast number of range-based volatility models that are known in the literature. Most of the models have been extended to include overnight jumps, which have also been proven to include valuable information on the change of price. Furthermore this research focusses on a short estimation horizon of only one single day. The price information available on day *t* is used for estimating the true volatility on day *t.* It is, however, trivial to estimate the volatility for a larger number of historical price observations or for a larger number of days. Each of the range-based volatility models can easily be extended to multiple days if required. A single day has been chosen to compare the efficiency of the range-based volatility estimators with the unbiased volatility estimator, i.e. the TTSE, which is also based on a single day. Hence, the price information available on day *t* is used to estimate the volatility of day *t*. The estimated daily volatility is benchmarked against the TTSE volatility model, which is a robust and unbiased estimator of the Integrated Volatility. A substantial price history of 6 East European stock market indices has been used for the empirical analysis.

In the first part of this research, sections 3 and 4, the theory of volatility analysis was discussed together with the challenges in estimating Realized Volatility. Various stylized facts have been addressed and mirrored against the stock market indices to analyse the impact on the empirical data set. Section 5 discusses the impact of overnight jumps and extends the existing range-based volatility models.

The second part of this research, section 6, discusses the theory of ranking volatility estimates. The broad financial literature on ranking volatility estimates is not unanimous on the ultimate ranking methodology, but rather proposes several ranking methodologies and no guidelines for practical use. Section 6 also proposes a new ranking methodology that is based on a Copula function approach, which aims to rank volatility models according to their performance during extreme movements. This is a new approach in ranking volatility estimates, which is of particular interest to various risk management functions. For robustness of the results, two additional and non-overlapping time periods have been included in the analysis.

Next to the analysis of the volatility estimators, the dissertation includes 2 applications of low-frequency range-based volatility estimators. These applications have been included to provide evidence of their applicability with practical examples of an application in Value-at-Risk and portfolio optimization.

To the best of our knowledge none of the previous research papers compared the results of various ranking methodologies and included the Tail correlation approach to rank the results. This research enriches the literature further with an empirical dataset consisting of intraday price observations comprising of 6 East European indices. The presented research gives an innovative and complete view on the performance of range-based volatility models on one side and ranking methodologies on the other side. The research also includes an application of the range-based volatility estimators with Value-at-Risk and an application with portfolio optimization.

The results show that both the Parkinson and the Garman and Klass models outperform the alternative range-based volatility models in most cases. Unlike the standard deviation or the daily squared return, which have, informally, become a market standard, the Parkinson and the Garman and Klass models show substantial better results across all the applied ranking methodologies. When overnight returns are available and contain information on price changes, it becomes beneficial to include this information in the range-based volatility model. In these cases the extended models that include overnight returns show overall better results than the same models that do not include the overnight price observations.

The choice of the ranking methodology largely depends on the purpose of the volatility estimates. For example, rankings based on the Coefficient of Efficiency, a Loss Function approach or a Mincer-Zarnowitz regression are all based on the overall performance of the range-based volatility models. The disadvantage is that the performance during severe stress is often underestimated. This is in particular a disadvantage when a volatility model is required that outperforms during periods of severe stress. In that case the Tail Dependence approach is recommended, because this methodology ranks the performance of the volatility estimators based on the historical extreme volatilities. The results of the Tail Dependence approach suggest in most cases a different range-based volatility model then when, for example, a loss function approach is used. When neither of the standard ranking methodologies provide a clear answer to the ranking question, the Tail Dependence approach can be consulted for advice.

The overall results of the ranking methodology suggest different range-based volatility models based on various ranking methodologies. Only the Garman-Klass, Parkinson, High-Low, Roger-Satchell and Yang-Zhang volatility models are suggested as the best volatility model depending on the ranking methodology and the period of time of the applied market. Based on the Coefficient of Efficiency either the Garman-Klass, Roger-Satchell or the Yang-Zhang model is suggested, while based on the Loss Function approach (either the MSE or the QLike function) or the Pearson's linear correlation ranking methodology, either the Parkinson or the Garman-Klass model is suggested across all markets and all time periods. When the Mincer-Zarnowitz regression or the Tail Dependence ranking methodology is applied, either the Garman-Klass, Parkinson or the High-Low volatility model is suggested across all markets. Thus in neither case there is a uniform result across the markets. However, the results show a clear pattern in which the Garman-Klass and Parkinson volatility models dominate across all markets and all periods of time. In most cases it is also beneficial to include overnight returns in the volatility estimator. One exception is the .WIG index which shows that the Garman-Klass model outperforms across all ranking methodologies and all periods of time.

The main hypothesis of this Thesis, H.1, states that amongst the range-based volatility estimators there is a least biased estimator of the Realized Volatility of stock market indices. In other words, range-based volatility estimators are appropriate models to estimate the 'true' volatility of stock indices. This hypothesis has been tested with the ranking methodologies described in chapter 6. The analysis in chapter 7 shows the results of the ranking exercises, while the application of the range based volatility estimators is shown in chapters 8 and 9. In total 5 different ranking methodologies have been applied to rank a wide set of volatility estimators. The volatility estimators that are non-range based include the daily squared return, the close-to-open estimator and the standard deviation. Throughout the ranking analysis neither of the 3 non-range based volatility estimators would have been suggested according to the ranking performance. Tables 7-21, 7-22 and 7-23 show a summary of the ranking results, which clearly indicate that in all ranking methodologies only the range based volatility estimators have been selected. It is clear from the results that either the Garman and Klass, Parkinson, High-Low, Yang-Zhang or the Roger and Satchell have been ranked highest. This result holds across all investigated indices and across all selected periods of time. The results of the two applications in Chapter 8 and 9 also provide additional evidence that support this hypothesis. The first application in section 8 tests various VaR models that utilize different volatility estimators. In almost all of the tested cases the High-Low volatility estimator could not be rejected based on the 1% or 5% significance level. The second application, shown in chapter 9, compares different portfolio optimization models and shows that the portfolio optimization model that utilizes the Parkinson range based volatility estimator outperforms the classical mean-variance model, which is based on the standard deviation, but it also outperforms the semi-variance portfolio optimization model. Given the various results of the ranking exercises and the two applications on VaR and portfolio optimization we find no evidence to reject the main hypothesis, H.1.

The first auxiliary hypothesis, H.1.1 states that range-based volatility estimators differ from each other. If this would not be the case it would mean that the selection process is redundant and one can simply choose any of the available range-based volatility estimators for estimating financial volatility. As a first step in analysing this hypothesis the properties of the range-based volatility estimators have been elaborated in chapter 3.2.2. Table 3-2 summarizes the model properties of the range-based volatility estimators. Not all range-based volatility estimators have the same properties and, in some cases, have unique properties. In addition to the elaboration of the properties described in chapter 3.2.2, the results of the ranking methodology in chapter 7 also provide no evidence that rejects the auxiliary hypothesis. The summary of the results presented in chapter 7.1 through 7.5 show that in most cases the ranking scores are unique. There is one exception with the High-Low (eq. 3-8) and Parkinson (eq. 3-9) volatility estimators. Since the only difference between the Parkinson and High-Low is a constant, the ranking results that are based on, e.g. correlation, can be equal in some cases. Hence, only in case of the High-Low and the Parkinson range-based volatility estimator this hypothesis is rejected. For the .Beti, .BUX, .PX and .SOFIX the ranking methodologies based on the Tail Dependence approach and the Pearson’s linear correlation functions show that when the High-Low is suggested as the best ranked model also the Parkinson is suggested as both ranking results are equal. Adding a constant to the High-Low volatility estimator gives no added value according to these ranking methodologies. Therefore following the Tail Dependence and the

Pearson’s linear correlation ranking methodologies both the High-Low and Parkinson are suggested. In terms of the hypothesis, H.1.1, the High-Low and Parkinson do not differ from each other when following the Tail Dependence or the Pearson’s linear correlation ranking methodology. Both approaches are based on the correlation between two distributions. However in all other cases we find no evidence to reject the hypothesis as the ranking results are unique.

The second auxiliary hypothesis, H.1.2, states that the efficiency of classical range-based volatility estimators can be increased by including overnight returns. Chapter 5.1 discusses the importance of the information carried in overnight returns across all 6 indices. The historical database, however, doesn’t include overnight returns for .Beti and .SOFIX, while Figure 5-1 provides evidence of overnight returns in the .BUX, .CROBEX, .PX and .WIG20 indices. Chapter 5.2 introduces the extended range-based volatility estimators, which extends the range of the classical range-based volatility estimators discussed in chapter 3.2.2 by simply including the overnight returns in the model. The results of the ranking methodologies in chapter 6 show that including overnight returns is in general beneficial for the volatility model. Evidence can be found in the ranking results based on the .BUX, .CRBX and PX stock market indices. In case of .WIG20 there is no support for the overnight returns, while in cases of .Bet and .SOFIX the overnight returns were not observed in the database and could therefore not be tested. Only .WIG20 provides evidence for rejecting the auxiliary hypothesis, H.1.2, while in all other cases there is no support based on the empirical results to reject the auxiliary hypothesis.

The third auxiliary hypothesis, H.1.3, states that range-based volatility estimators are less biased compared to the daily squared return or standard deviation. We use the results of the ranking analysis in section 7, as in none of the ranking methodologies either the standard deviation or the daily squared return have been selected as the best volatility estimator. Both the daily squared return as well as the standard deviation show poor performance in the ranking analysis. The results of the application with VaR in section 8 and the application in portfolio optimization in chapter 9 also provide support of the hypothesis as the standard deviation performed poor against various range-based volatility estimators. Given the thorough analysis we find no evidence to reject the auxiliary hypothesis.

The second main hypothesis, H.2, states that the dependence between the ‘true’ volatility and range-based volatility estimators is non-linear and manifests in positive dependence in the tails of the distribution. This hypothesis is tested with different volatility ranking methodologies. The Tail Dependence approach described in chapter 6.5 explains the basic idea behind this ranking methodology. Positive tail correlation coefficients indicate dependence in the tail of the distribution, which cannot be detected with the Pearson’s linear correlation function and neither with a loss function or a Coefficient of Efficiency approach. Positive tail correlation coefficients indicate non-linearity. The results of the Tail Dependence ranking methodology in chapter 7.5 show that the tail correlations are in the third quantile and can be considered significant. Ignoring this information might result in suboptimal range-based volatility estimates. Hence, we find no evidence to reject the second main hypothesis.

APPENDIX 1: DISTRIBUTION OF REALIZED VOLATILITY ESTIMATES

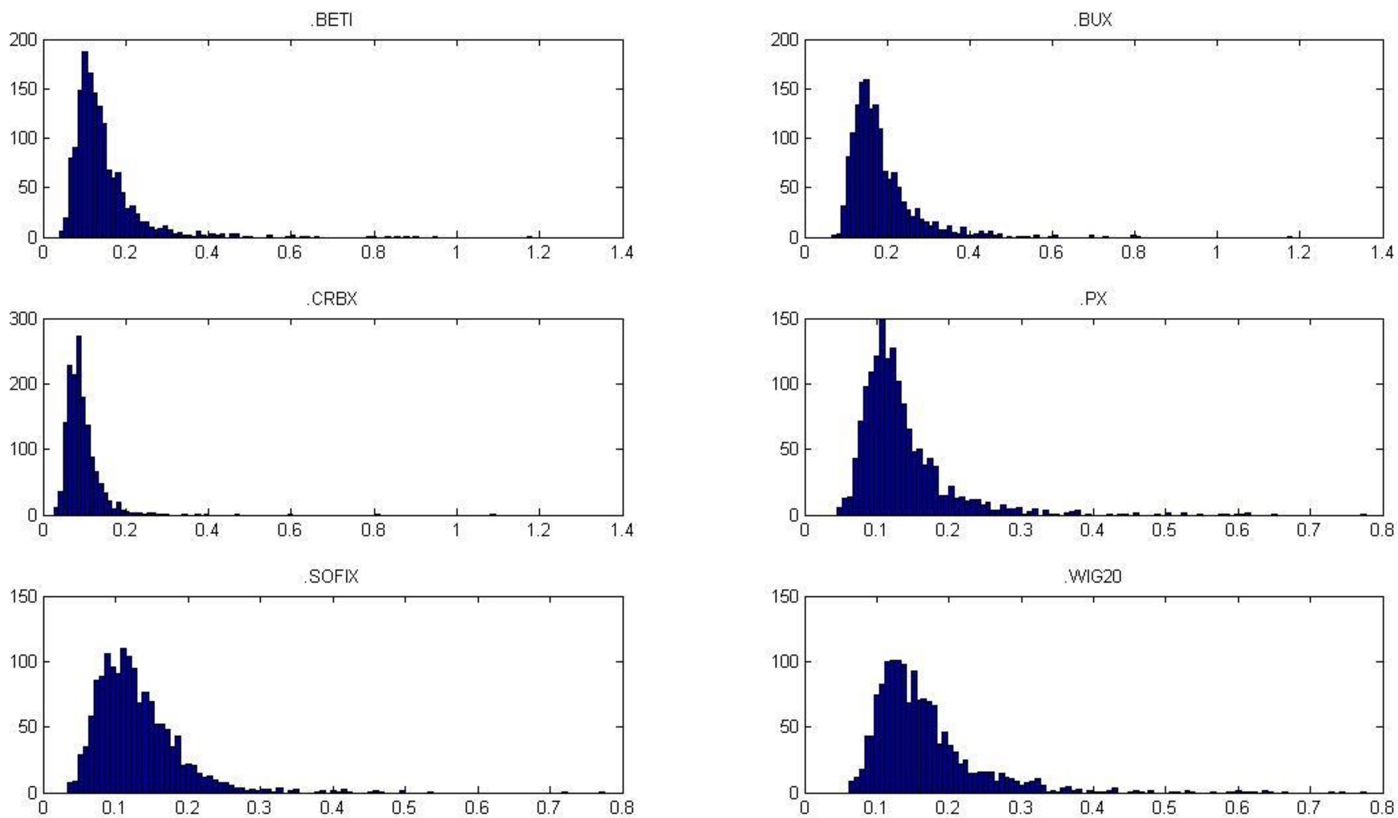


Figure A- 1 Distribution of Realized Volatility estimates based on 6 East European indices.

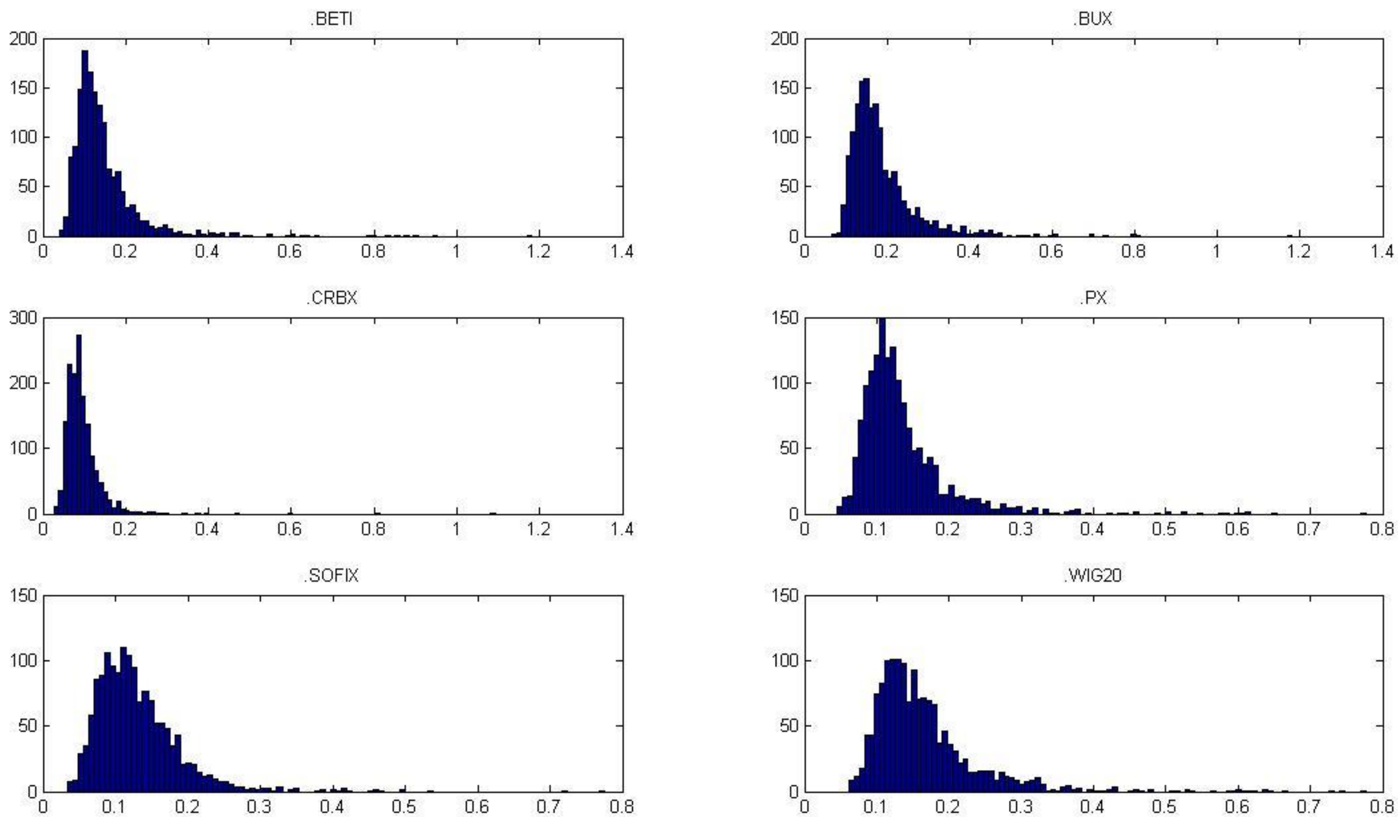


Figure A- 2 Distribution of Two Time Scale Volatility estimates based on 6 East European indices.

APPENDIX 2: DISTRIBUTIONS OF THE RANGE-BASED VOLATILITY ESTIMATES

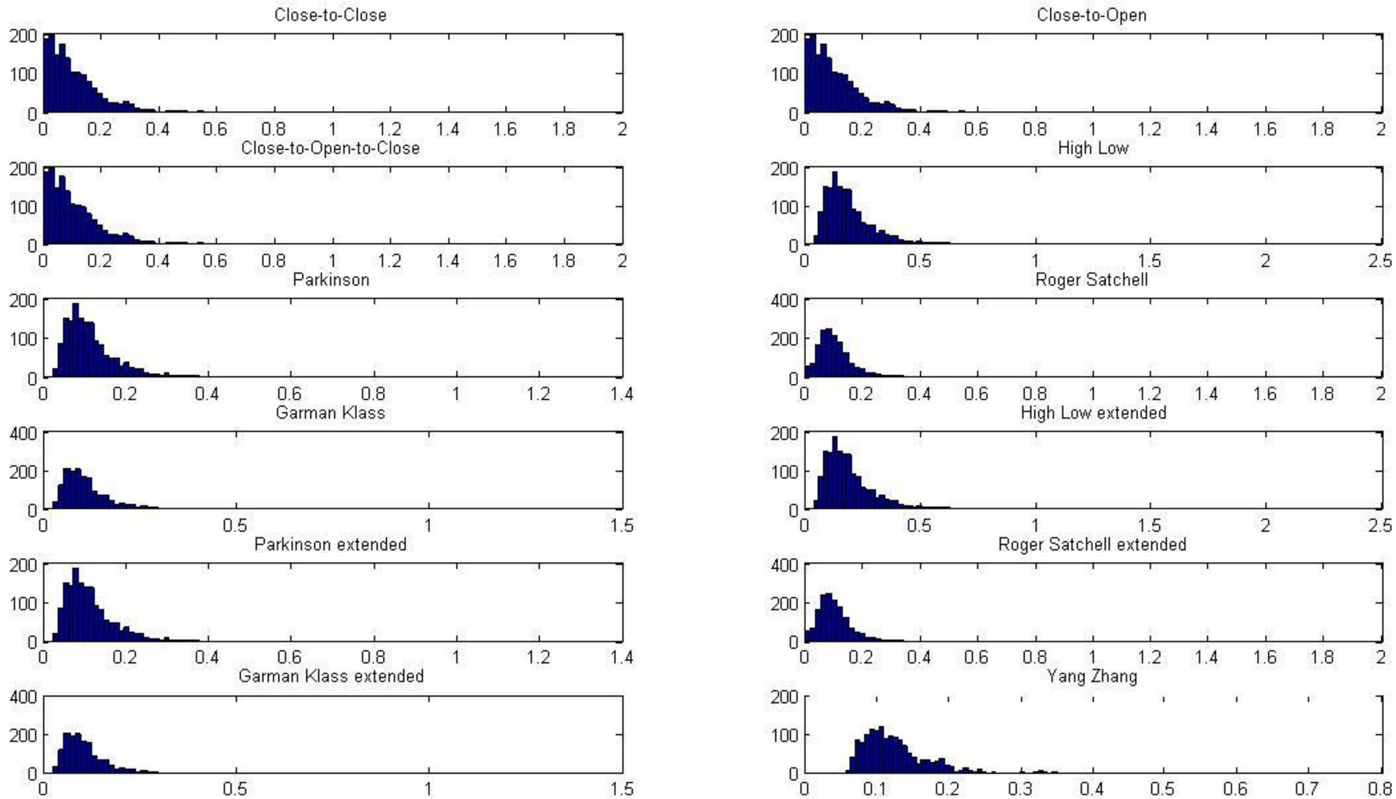


Figure A- 3 Distribution of volatility estimates based on the .BETI index

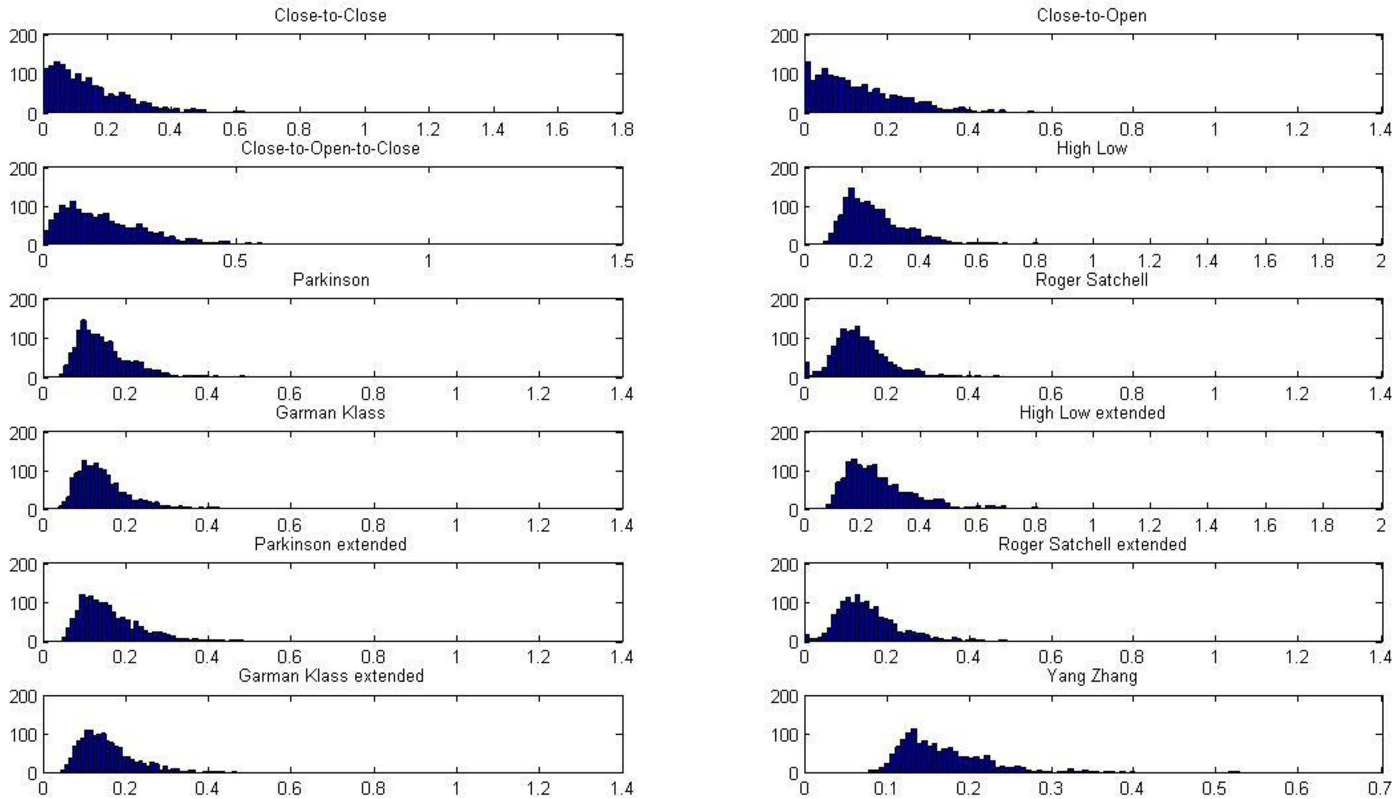


Figure A- 4 Distribution of volatility estimates based on the .BUX index

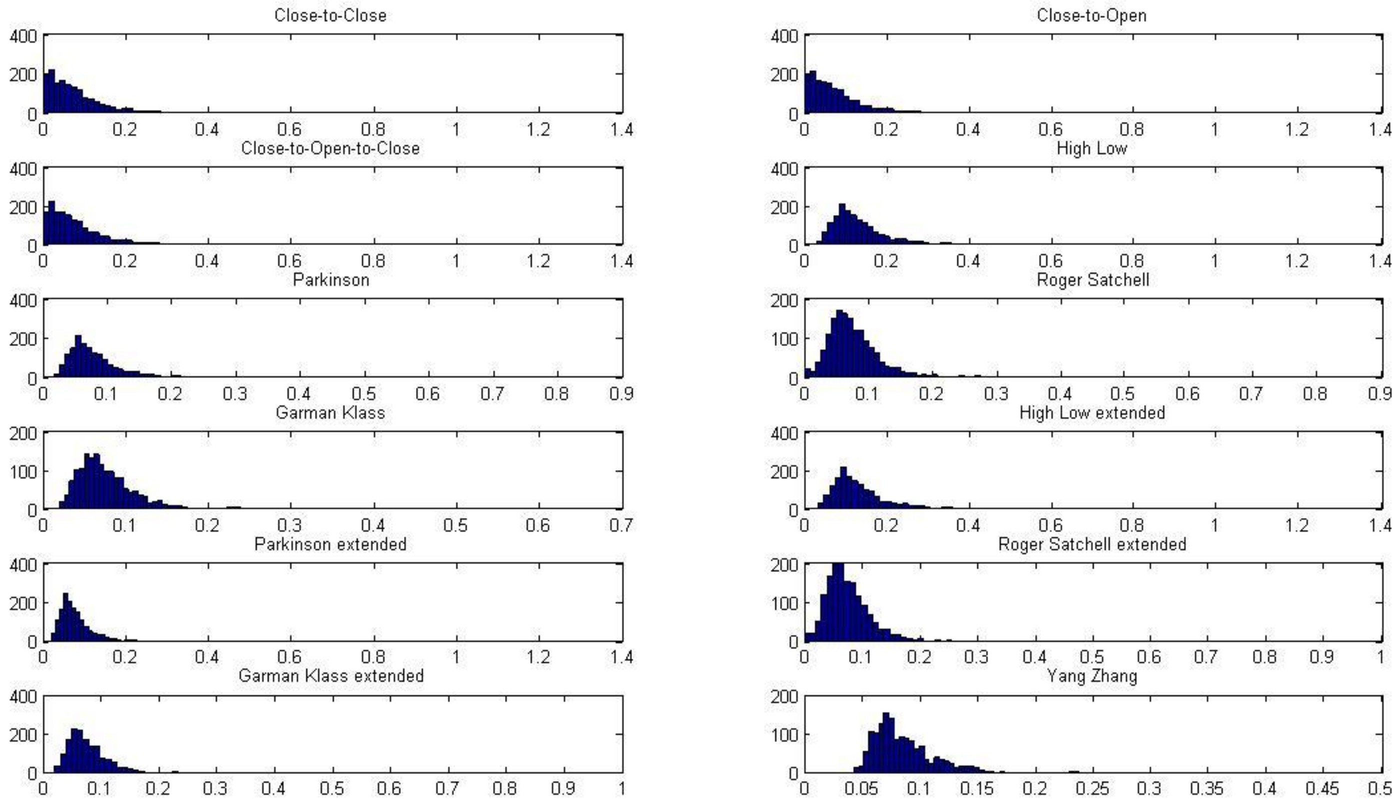


Figure A- 5 Distribution of volatility estimates based on the .CRBX index

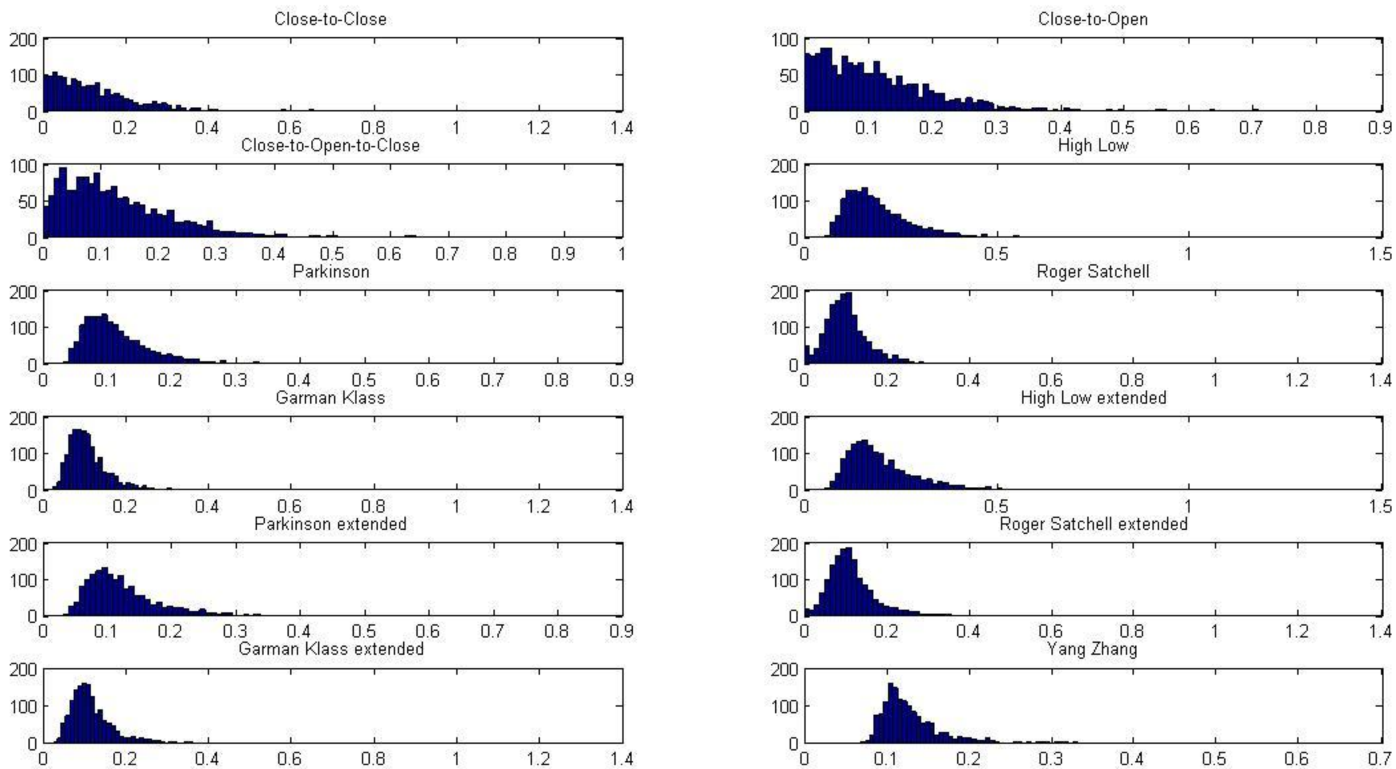


Figure A- 6 Distribution of volatility estimates based on the .PX index.

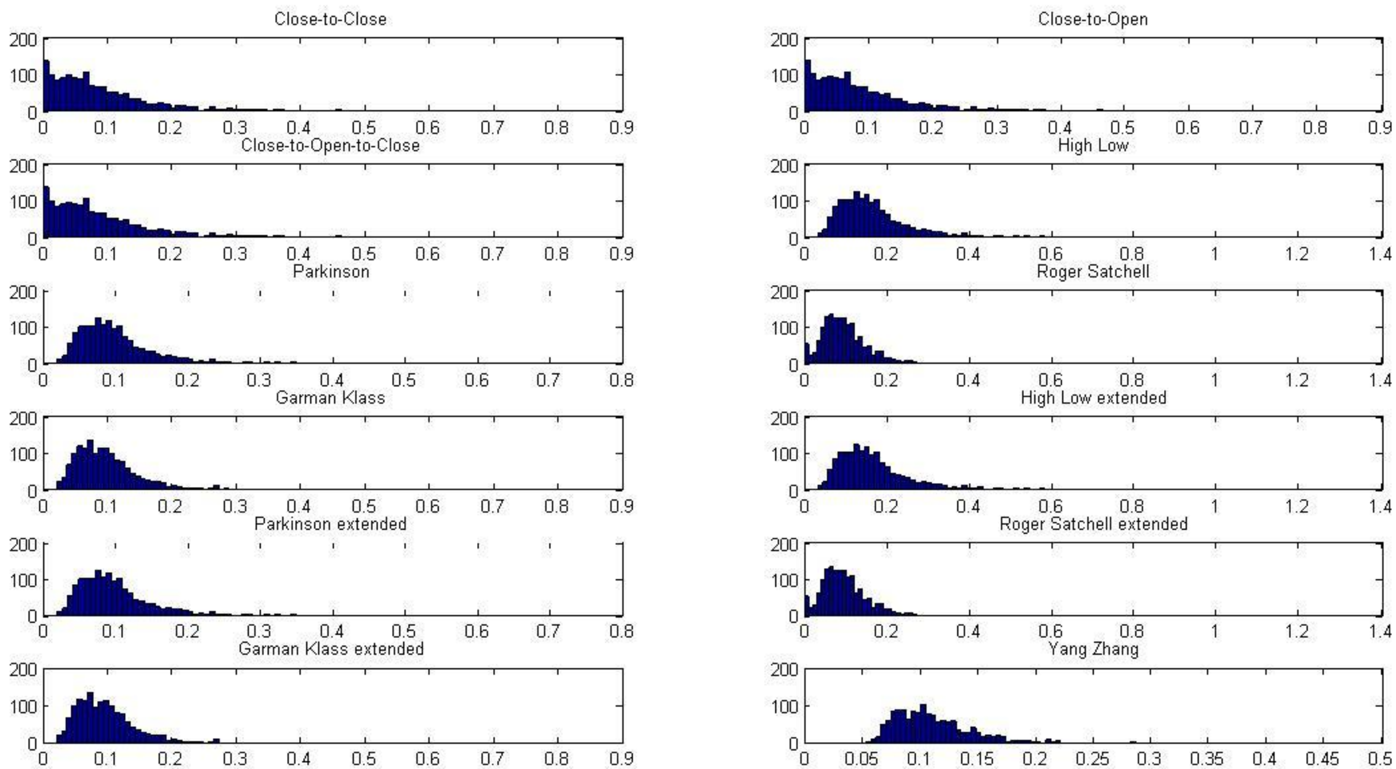


Figure A- 7 Distribution of volatility estimates based on the .SOFIX index.

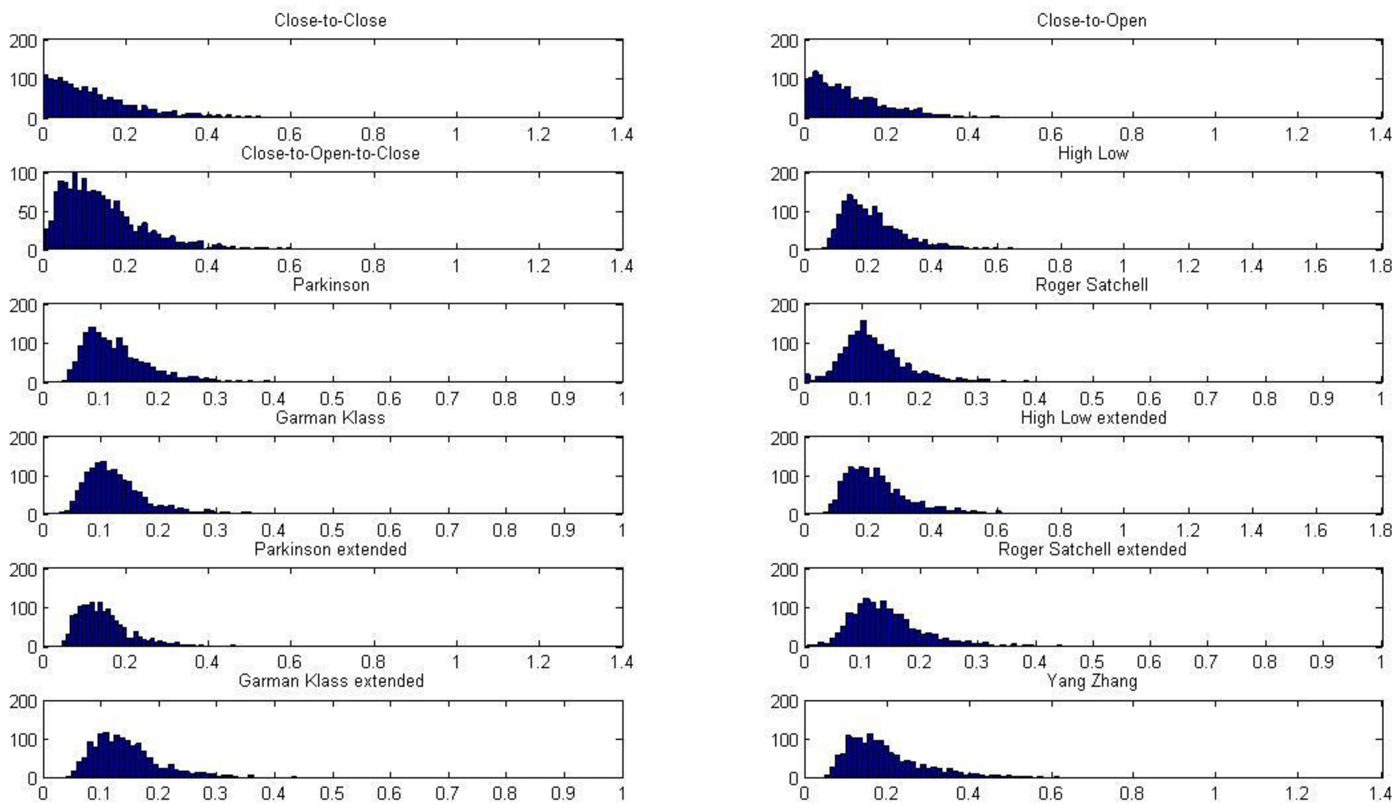


Figure A- 8 Distribution of volatility estimates based on the .WIG index.

APPENDIX 3: MINCER ZARNOWITZ REGRESSION RESULTS

Table A - 1. The Mincer-Zarnowitz regression results for the .BETI index

.BETI	4.1.2010 - 1.4.2016					4.1.2010 - 15.2.2013					15.2.2013 - 1.4.2016				
	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2
Daily	0.079	0.002	0.466	0.013	0.459	0.103	0.004	0.463	0.018	0.459	0.070	0.002	0.305	0.014	0.374
CO	0.079	0.002	0.466	0.013	0.459	0.100	0.004	0.464	0.018	0.459	0.070	0.002	0.305	0.014	0.374
COC	0.079	0.002	0.466	0.013	0.459	0.103	0.004	0.464	0.018	0.459	0.070	0.002	0.305	0.014	0.374
HL	0.022	0.002	0.541	0.007	0.788	0.031	0.003	0.562	0.009	0.827	0.041	0.002	0.348	0.010	0.607
Park	0.022	0.002	0.900	0.012	0.788	0.031	0.003	0.935	0.015	0.827	0.041	0.002	0.579	0.017	0.607
RS	0.045	0.003	0.792	0.018	0.541	0.062	0.004	0.824	0.026	0.566	0.055	0.002	0.458	0.020	0.384
GK	0.014	0.002	1.026	0.014	0.775	0.018	0.003	1.094	0.018	0.830	0.040	0.002	0.607	0.019	0.561
HL	0.022	0.002	0.541	0.007	0.788	0.031	0.003	0.562	0.009	0.827	0.041	0.002	0.348	0.010	0.607
Park*	0.022	0.002	0.900	0.012	0.788	0.031	0.003	0.935	0.015	0.827	0.041	0.002	0.579	0.017	0.607
RS*	0.045	0.003	0.792	0.018	0.541	0.062	0.004	0.824	0.026	0.566	0.055	0.002	0.458	0.020	0.384
GK*	0.014	0.002	1.026	0.014	0.775	0.018	0.003	1.094	0.018	0.830	0.040	0.002	0.607	0.019	0.561
YZ	0.020	0.004	0.840	0.023	0.462	0.035	0.006	0.837	0.033	0.449	0.052	0.004	0.419	0.035	0.152

Table A - 2. The Mincer-Zarnowitz regression results for the .BUX index.

.BUX	4.1.2010 - 1.4.2016					4.1.2010 - 15.2.2013					15.2.2013 - 1.4.2016				
	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2
Daily	0.105	0.002	0.253	0.011	0.266	0.126	0.004	0.234	0.015	0.229	0.090	0.002	0.236	0.013	0.301
CO	0.100	0.002	0.309	0.012	0.301	0.100	0.004	0.317	0.018	0.287	0.089	0.002	0.251	0.013	0.326
COC	0.092	0.002	0.317	0.011	0.349	0.106	0.004	0.313	0.017	0.313	0.086	0.002	0.256	0.013	0.327
HL	0.036	0.002	0.418	0.007	0.722	0.042	0.003	0.426	0.009	0.731	0.042	0.002	0.353	0.009	0.660
Park	0.036	0.002	0.696	0.011	0.722	0.042	0.003	0.710	0.015	0.731	0.042	0.002	0.587	0.015	0.660
RS	0.053	0.002	0.624	0.012	0.622	0.058	0.003	0.644	0.016	0.666	0.061	0.003	0.480	0.020	0.433
GK	0.032	0.002	0.741	0.011	0.752	0.039	0.003	0.743	0.015	0.765	0.037	0.002	0.653	0.017	0.656
HL	0.037	0.002	0.395	0.006	0.714	0.043	0.003	0.395	0.009	0.702	0.040	0.002	0.354	0.009	0.659
Park*	0.046	0.002	0.565	0.011	0.651	0.056	0.004	0.541	0.016	0.607	0.040	0.002	0.577	0.016	0.641
RS*	0.056	0.002	0.531	0.011	0.599	0.061	0.004	0.529	0.015	0.605	0.060	0.003	0.471	0.020	0.420
GK*	0.045	0.002	0.589	0.011	0.670	0.053	0.004	0.564	0.015	0.638	0.036	0.003	0.631	0.018	0.627
YZ	0.041	0.004	0.574	0.018	0.403	0.046	0.006	0.561	0.025	0.396	0.043	0.007	0.543	0.045	0.157

Table A - 3. The Mincer-Zarnowitz regression results for the .CRBX index

.CRBX	4.1.2010 - 1.4.2016					4.1.2010 - 15.2.2013					15.2.2013 - 1.4.2016				
	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2
Daily	0.058	0.002	0.467	0.015	0.372	0.064	0.003	0.491	0.023	0.378	0.060	0.001	0.273	0.018	0.234
CO	0.000	0.002	0.440	0.015	0.343	0.000	0.003	0.450	0.022	0.339	0.060	0.001	0.274	0.018	0.227
COC	0.063	0.002	0.397	0.015	0.313	0.073	0.003	0.392	0.022	0.297	0.060	0.001	0.279	0.018	0.231
HL	0.020	0.002	0.539	0.010	0.640	0.021	0.003	0.547	0.015	0.621	0.028	0.002	0.443	0.014	0.566
Park	0.020	0.002	0.897	0.017	0.640	0.021	0.003	0.911	0.025	0.621	0.028	0.002	0.737	0.023	0.566
RS	0.035	0.002	0.748	0.021	0.445	0.044	0.004	0.728	0.031	0.409	0.035	0.002	0.641	0.027	0.425
GK	0.016	0.002	0.977	0.020	0.613	0.018	0.003	0.984	0.030	0.582	0.022	0.002	0.839	0.025	0.588
HL	0.026	0.002	0.494	0.010	0.589	0.030	0.003	0.488	0.016	0.556	0.028	0.002	0.444	0.014	0.569
Park*	0.034	0.002	0.710	0.018	0.512	0.043	0.003	0.675	0.026	0.463	0.028	0.002	0.733	0.023	0.566
RS*	0.046	0.002	0.597	0.020	0.360	0.059	0.004	0.547	0.029	0.310	0.034	0.002	0.653	0.020	0.441
GK*	0.034	0.002	0.737	0.020	0.468	0.045	0.003	0.686	0.029	0.409	0.022	0.002	0.832	0.025	0.589
YZ	0.039	0.003	0.594	0.028	0.220	0.052	0.005	0.527	0.042	0.168	0.029	0.005	0.666	0.064	0.123

Table A - 4. The Mincer-Zarnowitz regression results for the .PX index.

.PX	4.1.2010 - 1.4.2016					4.1.2010 - 15.2.2013					15.2.2013 - 1.4.2016				
	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2
Daily	0.000	0.002	0.353	0.012	0.371	0.082	0.003	0.413	0.017	0.417	0.080	0.002	0.197	0.011	0.278
CO	0.000	0.002	0.297	0.015	0.195	0.000	0.004	0.358	0.025	0.204	0.081	0.002	0.200	0.012	0.271
COC	0.069	0.002	0.402	0.012	0.400	0.066	0.004	0.503	0.019	0.467	0.080	0.002	0.202	0.012	0.273
HL	0.037	0.003	0.433	0.011	0.485	0.041	0.004	0.499	0.017	0.537	0.047	0.002	0.291	0.010	0.537
Park	0.037	0.003	0.721	0.019	0.485	0.041	0.004	0.831	0.028	0.537	0.047	0.002	0.484	0.016	0.537
RS	0.059	0.003	0.598	0.020	0.362	0.069	0.004	0.686	0.028	0.429	0.063	0.002	0.379	0.020	0.307
GK	0.035	0.003	0.789	0.020	0.495	0.042	0.004	0.883	0.028	0.551	0.043	0.002	0.549	0.019	0.518
HL	0.025	0.002	0.472	0.009	0.637	0.024	0.003	0.537	0.012	0.712	0.047	0.002	0.291	0.010	0.537
Park*	0.024	0.002	0.745	0.011	0.729	0.023	0.003	0.808	0.015	0.787	0.046	0.002	0.485	0.016	0.538
RS*	0.039	0.002	0.690	0.013	0.653	0.040	0.003	0.749	0.016	0.741	0.061	0.002	0.390	0.020	0.320
GK*	0.023	0.002	0.793	0.012	0.749	0.026	0.003	0.832	0.015	0.802	0.042	0.002	0.551	0.010	0.522
YZ	0.025	0.003	0.709	0.023	0.388	0.036	0.005	0.688	0.031	0.381	0.037	0.005	0.544	0.041	0.183

Table A - 5. The Mincer-Zarnowitz regression results for the .SOFIX index.

.SOFIX	4.1.2010 - 1.4.2016					4.1.2010 - 15.2.2013					15.2.2013 - 1.4.2016				
	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2
Daily	0.087	0.002	0.417	0.014	0.349	0.093	0.003	0.374	0.017	0.371	0.080	0.003	0.469	0.024	0.333
CO	0.000	0.002	0.416	0.014	0.348	0.000	0.003	0.373	0.017	0.369	0.080	0.003	0.469	0.024	0.333
COC	0.087	0.002	0.417	0.014	0.349	0.093	0.003	0.374	0.017	0.370	0.080	0.003	0.469	0.024	0.333
HL	0.035	0.002	0.521	0.000	0.739	0.042	0.002	0.471	0.010	0.720	0.026	0.002	0.581	0.012	0.764
Park	0.035	0.002	0.868	0.013	0.739	0.042	0.002	0.784	0.017	0.720	0.026	0.002	0.968	0.019	0.764
RS	0.053	0.002	0.725	0.015	0.599	0.061	0.003	0.654	0.021	0.547	0.046	0.003	0.796	0.021	0.645
GK	0.032	0.002	0.919	0.010	0.742	0.038	0.002	0.845	0.010	0.717	0.026	0.002	1.002	0.020	0.767
HL	0.035	0.002	0.521	0.000	0.739	0.042	0.002	0.471	0.010	0.720	0.026	0.002	0.581	0.012	0.764
Park*	0.035	0.002	0.866	0.013	0.738	0.043	0.002	0.781	0.017	0.718	0.026	0.002	0.968	0.019	0.764
RS*	0.053	0.002	0.723	0.010	0.597	0.061	0.003	0.651	0.021	0.544	0.046	0.003	0.796	0.020	0.645
GK*	0.032	0.002	0.916	0.010	0.739	0.039	0.002	0.840	0.019	0.714	0.026	0.002	1.002	0.020	0.767
YZ	0.047	0.004	0.686	0.031	0.244	0.056	0.006	0.615	0.044	0.197	0.040	0.005	0.738	0.044	0.275

Table A - 6. The Mincer-Zarnowitz regression results for the .WIG index.

.WIG	4.1.2010 - 1.4.2016					4.1.2010 - 15.2.2013					15.2.2013 - 1.4.2016				
	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2	Intercept	s.e.	Volatility	s.e.	R^2
Daily	0.100	0.002	0.246	0.011	0.233	0.104	0.003	0.267	0.017	0.244	0.097	0.002	0.202	0.014	0.207
CO	0.000	0.002	0.334	0.012	0.325	0.000	0.003	0.361	0.018	0.338	0.091	0.002	0.282	0.015	0.302
COC	0.084	0.002	0.328	0.012	0.342	0.085	0.004	0.353	0.017	0.361	0.086	0.003	0.272	0.016	0.284
HL	0.034	0.002	0.441	0.006	0.749	0.034	0.002	0.454	0.009	0.783	0.038	0.002	0.405	0.010	0.670
Park	0.034	0.002	0.734	0.011	0.749	0.034	0.002	0.756	0.014	0.783	0.038	0.002	0.674	0.017	0.670
RS	0.047	0.002	0.665	0.013	0.640	0.049	0.003	0.685	0.016	0.695	0.051	0.003	0.599	0.021	0.513
GK	0.030	0.002	0.780	0.011	0.770	0.031	0.002	0.791	0.014	0.809	0.032	0.002	0.738	0.018	0.677
HL	0.034	0.002	0.412	0.007	0.715	0.033	0.003	0.424	0.009	0.751	0.039	0.003	0.381	0.011	0.624
Park*	0.044	0.002	0.565	0.011	0.611	0.043	0.003	0.578	0.015	0.642	0.048	0.003	0.529	0.018	0.518
RS*	0.050	0.002	0.541	0.012	0.567	0.048	0.003	0.559	0.016	0.622	0.056	0.003	0.489	0.021	0.421
GK*	0.042	0.002	0.586	0.012	0.619	0.041	0.003	0.595	0.015	0.659	0.045	0.003	0.556	0.020	0.506
YZ	0.075	0.003	0.275	0.010	0.320	0.076	0.004	0.292	0.014	0.372	0.081	0.004	0.217	0.016	0.188

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